

Please put your work on this page. Box your answers. Thanks and enjoy. You have 6 minutes to complete this quiz.

Problem 1 [6pts] Suppose $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $X = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ and $v = [1, 2]^T$. Calculate the following:

1. $A + X$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} 2 & 2 \\ 2 & 6 \end{bmatrix}}$$

2. $AX = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & 4 \\ -1 & 8 \end{bmatrix}}$

3. $Av = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1+4 \\ 3+8 \end{bmatrix} = \boxed{\begin{bmatrix} 5 \\ 11 \end{bmatrix}}$

Problem 2 [4pts] Suppose $x'' + y + 3 = 0$ and $y' = x - y$. Introduce variables x_1, x_2, x_3 to rewrite the given system as a system of ODEs in normal form. Explicitly define x_1, x_2, x_3 and be sure to find the matrix A for which the system becomes $\frac{d\vec{x}}{dt} = A\vec{x} + \vec{f}$ where $\vec{x} = [x_1, x_2, x_3]^T$

$$\begin{aligned} x_1 &= x &\rightarrow x_1' &= x_3 \\ x_2 &= y &\rightarrow x_2' &= y' = x - y = x_1 - x_2 \\ x_3 &= x' &\rightarrow x_3' &= x'' = -y - 3 = -x_2 - 3 \end{aligned}$$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}' = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix}}$$

Other answers possible, could define x_1, x_2, x_3 differently.