MATH 334: SPRING 2012:	QUIZ 7

Please put your work on this page. Box your answers. Thanks and enjoy. You may compare answers, but do not work together. This is due before 4:45pm tomorrow (Friday) in my drop-box on the 4-th floor DeMoss. Keep in mind the math suite locks at 5pm and it may not be possible to drop it off after 5pm.

Problem 1 [4pts] Suppose $y'' + \frac{x}{(x-2)(x^2-6x+10)}y' + \left(\frac{1}{(x+3)^3} + \frac{1}{x^2}\right)y = 0.$

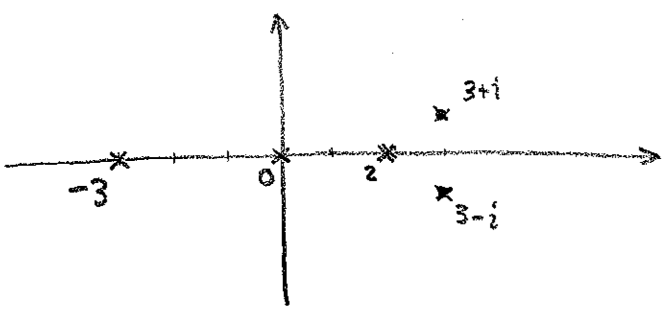
$(x-3)^2+1 \implies 3 \pm i$ sing. pts.

1. find all singular points
2. classify each real singular point as either regular or irregular (not regular)
3. plot the singularities in a complex plane
4. find the largest possible open and real domain of the solution $y = \sum_{n=0}^{\infty} a_n(x-0.5)^2$
5. find the largest possible open and real domain of the solution $y = \sum_{n=0}^{\infty} a_n(x-4)^2$

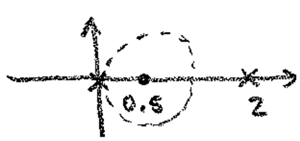
1.) we have division by zero in the coefficient fncts at $x = 2,\ 3+i,\ 3-i,\ -3,\ 0.$

2.) $x=2$, $x=0$ are both regular singular pts. ($(x-2)^2 L(x)$ & $x^2 L(x)$ have analytic coefficients.)
$x=-3$ is an irregular singular pt. ($(x-3)^3 L(x)$ still has $\frac{1}{x-3}$ term in the coefficients of y.)

3.)



4.) use plot in (3.) for guidance

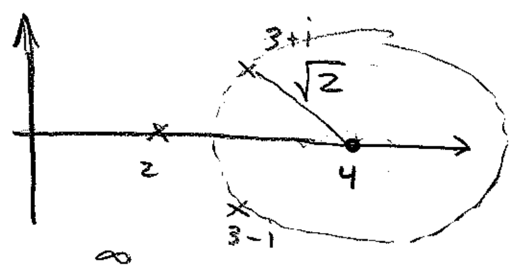


$y = \sum_{n=0}^{\infty} a_n(x-0.5)^2$ power series, centered at $x=0.5$

$\implies \boxed{(0,1)}$

(could be $(0,1]$, but I asked for open to avoid this ambiguity)

5.)



$y = \sum_{n=0}^{\infty} a_n(x-4)^2$ is power series centered at $x=4$ (an ordinary pt.) hence largest open is

$\boxed{(4-\sqrt{2},\ 4+\sqrt{2})}$

(due to proximity of $3 \pm i$ singularities)

Problem 2 [3pts] Suppose $y(0) = 1$ and $y'(0) = 2$. Find the solution up to order 5 in x for the differential equation

$$y'' + (x^2 - 1)\cos(x)y' + \sinh(3x)y = 0.$$

Let $y = c_0 + c_1 x + c_2 x^2 + c_3 x^4 + c_5 x^5 + \cdots$

Note $y(0) = c_0 = 1$ and $y'(0) = c_1 = 2 \Rightarrow y = 1 + 2x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + \cdots$

We find, $y' = 2 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + \cdots \quad y'' = 2c_2 + 6c_3 x + 12 c_4 x^3 + \cdots$

Thus,

$$2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + (x^2-1)\left(1 - \tfrac{1}{2}x^2\right)\left(2 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3\right) +$$

$$+ \left(3x + \tfrac{1}{6}(3x)^3\right)\left(1 + 2x + c_2 x^2\right) + \cdots = 0$$

I've kept up to order x^3 terms.

Const) $2c_2 - 2 = 0 \Rightarrow c_2 = 1.$

x) $6c_3 - 2c_2 + 3 = 0 \Rightarrow c_3 = -1/6.$

x^2) $12c_4 + 2 + 1 - 3c_3 + 6 = 0 \Rightarrow c_4 = \frac{1}{12}\left(-9 - \frac{3}{6}\right) = \frac{-19}{24} = \frac{-19}{24} = c_4$

x^3) $20c_5 + 2c_2 + c_2 - 4c_4 = 0 \Rightarrow c_5 = \frac{1}{20}\left(-3 - \frac{76}{24}\right) = \frac{1}{20}\left(\frac{-148}{24}\right) = \frac{-37}{120}$

$$\boxed{y = 1 + 2x + x^2 - \frac{1}{6}x^3 - \frac{19}{24}x^4 - \frac{37}{120}x^5 + \cdots}$$

Problem 3 [3pts] Find the complete power series solution centered at $x_o = 0$ for $y'' - 9y = 0$. **please use the series technique, I know other methods are easier, I'm trying to be nice about this problem, don't make me regret it!**

Remark:
cosh(

$$y = \sum_{n=0}^{\infty} c_n x^n$$

$$y' = \sum_{n=1}^{\infty} n c_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} = \sum_{j=0}^{\infty} (j+2)(j+1)c_{j+2} x^j$$

$$\Rightarrow y'' - 9y = \sum_{n=0}^{\infty} \left((n+2)(n+1)c_{n+2} - 9c_n\right)x^n$$

Hence $c_{n+2} = \dfrac{9c_n}{(n+2)(n+1)}$

$$c_2 = \frac{9c_0}{2\cdot 1}, \quad c_4 = \frac{9c_2}{4\cdot 3} = \frac{9^2 c_0}{4\cdot 3\cdot 2\cdot 1}, \ldots \quad c_{2n} = \frac{9^n c_0}{(2n)!}$$

Likewise, $c_{2n+1} = \dfrac{9^n c_1}{(2n+1)!}$. We find

$$\boxed{y = c_0\left(\sum_{n=0}^{\infty} \frac{9^n}{(2n)!}x^{2n}\right) + c_1\left(\sum_{n=0}^{\infty} \frac{9^n}{(2n+1)!}x^{2n+1}\right) = c_0 \sum_{n=0}^{\infty} \frac{(3x)^{2n}}{(2n)!} + \frac{c_1}{3}\left(\sum_{n=0}^{\infty} \frac{(3x)^{2n+1}}{(2n+1)!}\right) = c_0\cosh(3x) + \frac{c_1}{3}\sinh(3x)}$$