Please put your work on these pages. Box your answers. Thanks and enjoy. Problems 1,2 and 3 are basic problems which ought not involve a substitution (except, perhaps, to do an integral). However, Problem 4 is a Bernoulli equation as we discussed in lecture. Beyond that, it's up to your imagination.

**Problem 1** [20pts] Find the explicit solution of $\frac{dy}{dx} = y$ through $(0, -1)$.

**Problem 2** [20pts] Find the explicit solution of $\frac{dy}{dx} = \sqrt{\frac{y+1}{x-1}}$ through $(2, 3)$.

**Problem 3** [20pts] Find the explicit general solution of $\frac{dy}{dx} + \frac{2y}{x} = e^{x^3}$.

**Problem 4** [30pts] Find the general implicit solution of $(2xy^2 + e^x)dx + (2x^2y - \sin(y))dy = 0$.

**Problem 5** [20pts] Find the explicit general solution of $\frac{dy}{dx} + \frac{3y}{x} = xy^2$.

**Problem 6** [20pts] Solve $\frac{dy}{dx} = (x + y + 3)^2$.

**Problem 7** [20pts] Suppose a tank of salty water has $15kg$ of salt disolved in $1000L$ of water at time $t = 0$. Furthermore, assume pure water enters the tank at a rate of $10L/min$ and salty water drains out at a rate of $10L/min$. If $y(t)$ is the number of $kg$ of salt at time $t$ then **find** $y(t)$ **for** $t > 0$. We suppose that this tank is arranged such that the concentration of salt is constant throughout the liquid in this mixing tank.

**Problem 8** [10pts] Let $a, b$ be particular positive constants. Find the orthogonal trajectories to the ellipse $x^2/a^2 + y^2/b^2 = 1$.

**Problem 9** [20pts] Suppose a rocket car has an initial speed of $v_o$ as it hurtles across a speedway in a remote desert. Suppose the driver opens a parachute which developes a retarding force proportional to the cube of the velocity; $F_f = -kv^2$. Find the velocity as:

(a.) a function of time,

(b.) a function of position $x$ taking $x_o$ as the initial position

**Problem 10** [10pts] Suppose $\frac{dx}{dt} = (x + y)^{42}$ and $\frac{dy}{dt} = cos(x + y) - \sqrt{x^2 + y^2}$. Give the first order ODE in Pfaffian form whose solutions are parametrized by the solutions of the given system of ODEs. **DO NOT ATTEMPT A SOLUTION OF THIS DEQn !**

**Problem 11** [10pts] Let $\star$ be the DEqn $y^4 \cos(4x)dx + y^3 f(x)dy = 0$. Find all functions $f$ such that $\star$ is an exact DEqn.

**Problem 12** [10pts] Suppose $\frac{dy}{dx} = \cos(x - y)$. Discuss the plotted solutions given below. In particular, comment on the uniqueness theorem and any apparent singular solutions.

**Bonus** Suppose $Mdx + Ndy = 0$ has solution $F(x, y) = k$. Moreover, suppose that $Mdx + Ndy = 0$ is **not** exact. Use the given solution to find an integrating factor $I$ such that $IMdx + INdy = dG$ for some function $G$.