YOUR NAME HERE:

MATH 334	TEST III

This test is open book and you must show work for full credit. The only person you are allowed to get help with on this is me. You can consult static sources (books, non-response internet pages, wiki's etc...) and use Mathematica and/or Matlab etc... to solve integrations and/or produce plots. This is due on the reading day by 4:30pm. (remember the math suite closes at 5pm so getting it in on time is important, I will take off points for lateness here)

bonus: animate the solutions from either 9 or 10. Worth 20pts

Problem 1 [20pts] Find the first two terms in each of the Frobenius-type solutions of $4xy''+\frac{1}{2}y'+y=0$ at the regular singular point $x=0$. This means your general solution should have terms of up to (but not including) x^2.

Problem 2 [20pts] Solve $x' = 2x + y + 1$, $y' = 2y + z + 1$ and $z' = 2z + 1$ by whatever method you prefer.

Problem 3 [20pts] Solve $x'' + 10x - 4y = 0$ and $-4x + y'' + 4y = 0$ subject the initial conditions $x(0) = 0, x'(0) = 1$ and $y(0) = 0$ and $y'(0) = -1$. *Hint: you could use Laplace transforms here, you'd get a system in X and Y to unravel... or you could use the method of Chapter 5 we dicussed right after Test 2, but then you have to fit initial conditions... your choice*

Problem 4 [20pts] Solve $x' = x - 2y$ and $y' = 5x - y$ by the eigenvector method.

Problem 5 [20pts] Solve $y''+4y'+13y = \delta(t-\pi)+\delta(t-3\pi)$ with initial conditions $y(0) = 1, y'(0) = 0$.

Problem 6 [20pts] Let $f(t) = \begin{cases} t & 0 < t < 1 \\ \sin(t) & t > 1 \end{cases}$. Suppose $y(0) = y'(0) = 0$. Solve $y'' = f(t)$.

Problem 7 [20pts] Find the inverse Laplace transform of

$$F(s) = \frac{3se^{-2s}}{s^3 + 8s^2 + 20s}$$

Problem 8 [20pts] Suppose $B(t) = \begin{bmatrix} 1+t & 4t \\ 2t & 1-2t \end{bmatrix}$.

1. calculate $B'(t)$
2. calculate $B'(0)$
3. let A be a square matrix, show that $\frac{d}{dt}\left[e^{At}\right]\Big|_{t=0} = A$.
4. show that if $Av = \lambda v$ then $e^{At}\vec{v} = e^{\lambda t}\vec{v}$
5. is it possible that $B(t) = e^{tA}$ for some matrix A?

you can work together on Problems 9 and 10, but NOT the other problems! This you may recognize as Problem 69 and 71 from Problem Set IV)

Problem 9 [20pts] Solve the heat equation $u_t = u_{xx}$ on $0 < x < \pi$ for $t > 0$ subject to the boundary conditions $u_x(0,t) = u_t(\pi,t) = 0$ for $t > 0$ and the initial condition $u(x,0) = e^x$ for $0 < x < \pi$.

Problem 10 [20pts] Solve $u_{tt} = u_{xx}$ for $0 < x < 1$ and $t > 0$ subject the boundary conditions $u(0,t) = u(1,t) = 0$ for $t > 0$ given the initial conditions $u(x,0) = x(1-x)$ and $u_t(x,0) = \sin(5\pi x) + \sin(10\pi x)$.

you may use the general solution given on PH-146-147, the solution of 10.6 number 1 of Nagel Saff and Snider. You just need to understand the solution well enough to slightly modify it. You can just outline the calculation, all the details need not be repeated. Do box your answer as always!