Make sure you name is on each page and the assignment is stapled. Thanks and enjoy.

Problem 1 Find power series expansion for $f(x) = x^2 \sin(x^3)$ centereed at $x_o = 0$. Do not do this by Taylor's Theorem directly!

Problem 2 exercise 1 of page 438 in text.

Problem 3 exercise 16 of page 439 in text.

Problem 4 exercise 23 of page 440 in text.

Problem 5 exercise 24 of page 440 in text.

Problem 6 exercise 32 of page 440 in text.

Problem 7 exercise 13 of page 450 in text.

Problem 8 exercise 22 of page 450 in text.

Problem 9 exercise 25 of page 450 in text.

Problem 10 exercise 17 of page 456 in text.

Problem 11 Find the power series expansion for $f(x) = x\sin(x)$ centered at $x_o = 3$.

Problem 12 Find the power series expansion of $f(x) = \frac{x^2}{2+x}$ centered at $x_o = -1$.

Problem 13 Define $e^x = \sum_{n=0}^{\infty} \frac{1}{n!}x^n$ and likewise for e^y. Prove $e^x e^y = e^{x+y}$ by multiplying the series for e^x and e^y. Use the Cauchy product defined on pg. 434 of your text to multiply the series.

Problem 14 Suppose $\sum_{k=0}^{\infty}(a_{2k}x^{2k} + b_{2k+1}x^{2k+1} = e^x + \cos(x+2)$. Find explicit formulas for a_{2k} and b_{2k+1} via Σ-notation algebra.

Problem 15 Find a power series solution to the integrals below:

(a.) $\int \frac{x^3+x^6}{1-x^3}\, dx$

(b.) $\int x^8 e^{x^3+2}\, dx$

Problem 16 Calculate the 42^{nd}-derivative of $x^2\cos(x)$ at $x = 1$. (use power series techniques)

Problem 17 Find the complete power series solution of $y'' + x^2y' + 2xy = 0$ about the ordinary point $x = 0$. Your answer should include nice formulas for arbitrary coefficients in each of the fundamental solutions. You need to both set-up and solve the reccurrence relations as best you can.

Problem 18 (Ritger & Rose 7-2 problem 7 part c) Find the first four nonzero terms in the power series solution about zero for the initial value problem $(x+2)y'' + 3y = 0$ with $y(0) = 0$ and $y'(0) = 1$.

Problem 19 (Ritger & Rose 7-2 problem 7 part d) Find the first four nonzero terms in the power series solution about zero for the initial value problem $y'' + \sin(x)y' + (x-1)y = 0$ with $y(0) = 1$ and $y'(0) = 0$.

Problem 20 Construct a differential equation with $y_1(x) = \frac{\sin(x)}{x}$ for $x \neq 0$ and $y_1(0) = 1$, $y_2(x) = x$ as its fundamental solution set. To accomplish this task do two tasks:

(a.) Argue from appropriate facts from the theory of determinants that $L[y] = \det \begin{bmatrix} y & y' & y'' \\ y_1 & y_1' & y_1'' \\ y_2 & y_2' & y_2'' \end{bmatrix}$ is a linear ODE with solutions y_1 and y_2.

(b.) calculate $L[y]$ explicitly as a linear ODE of the form $py'' + qy' + ry = 0$ where p, q, r are perhaps given as Taylor expansions about zero. (just find the first few terms in the taylor expansions of the coefficient functions p, q, r)