MATH 334:	MISSION 6

Please put your work on these sheets. If you need additional room to show your work then add paper as needed, but be sure to put your answer clearly near the problem statement. Box your answers. Make sure you name is on each page and the assignment is stapled. Thanks and enjoy.

Problem 1 (3pts) Find the singular points of the following differential equations: (taken from Ritger & Rose section 7-2 problem 2)

(a.) $y'' + xy' + 3y = 0$

(b.) $(x^2 - 3x + 2)y'' + \sqrt{x}y' + x^2y = 0$

(c.) $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$

(d.) $(x^2 - x)y'' + x^2y' - 3xy = 0$

(e.) $e^x - 1)y'' + xy = 0$

(f.) $x(x^2 + 2x + 2)y'' + (x^2 + 1)y' + 3y = 0$

Problem 2 (5pts) Find the complete Frobenius solution of

$$x^2y'' + x(x - \tfrac{1}{2})y' + \tfrac{1}{2}y = 0.$$

(it turns out this one has real exponents)

Problem 3 (4pts) Find the Frobenius solution near $x = 0$ for $x > 0$ up to terms of order x^2 for

$$x^2y'' + \sin(x)y' - \cos(x)y = 0.$$

Problem 4 (3pts)(this is Problem 41 of section 8.6 of the 5-th Ed. of Nagle, Saff and Snider) Solve $x^3y'' - x^2y' - y = 0$ for $x >> 0$ by making the subsitution $z = 1/x$ and solving the resulting differential equation in z about the regular singular point $z = 0$. Find the first four nonzero terms in the series expansion about infinity.

and now for something completely different... these should be a welcome relief from the trouble of Frobenius

Problem 5 (2pts) Suppose $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$. Calculate A^2.

Problem 6 (2pts) Let A be as in the previous problem. Suppose $v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$.

(a.) calculate Av_1 c

(b.) calculate Av_2

(c.) calculate $A[v_1|v_2]$ (here $[v_1|v_2]$ is the 3×2 matrix made from gluing (aka concatenating) the column vectors v_1 and v_2)

(d.) Does $A[v_1|v_2] = [Av_1|Av_2]$?

Problem 7 (2pts) A square matrix X is invertible iff there exists Y such that $XY = YX = I$ where I is the identity matrix. Moreover, linear algebra reveals that X is invertible iff $det(X) \neq 0$. For a 2×2 matrix $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ we define $det(X) = ad - bc$. Suppose X is invertible and show $X^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. This formula is worth memorizing for future use in two-dimensional problems. Please understand, all I'm asking here is for you to multiply X and my propsed formula for X^{-1} to obtain $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Problem 8 (2pts) Differentiation of matrices of functions is not hard. Let $X(t) = \begin{bmatrix} e^t & t \\ 1/t & e^{-t} \end{bmatrix}$. Calculate:

(a.) calculate $\frac{dX}{dt}$

(b.) calculate $\frac{dX^{-1}}{dt}$

(c.) simplify $\frac{dX}{dt}X^{-1} + X\frac{dX^{-1}}{dt}$.

(d.) explain the previous part by differentiating $X(t)X^{-1}(t) = I$. Note: the product rule for matrix products is simply $\frac{d}{dt}(AB) = \frac{dA}{dt}B + A\frac{dB}{dt}$.

Problem 9 (2pts) Reformulate the system of differential equations $y'' + z = 0$ and $z'' - y = 0$ as a system of four first order linear differential equations via the substitution $y = x_1, y' = x_2, z = x_3$ and $z' = x_4$. Write the system of first order differential equations in matrix form.