Make sure you name is on each page and the assignment is stapled. Thanks and enjoy. These problems are worth 2pts a piece (this makes 40 total points)

Problem 1 Calculate the Laplace transforms of $f(t) = \sin(t)\cos(2t) + \sin^2(3t)$.

Problem 2 Calculate the Laplace transforms of $f(t) = e^t u(t-3) + \sin(t)u(t-6)$.

Problem 3 Calculate the Laplace transforms of the following function:

$$f(t) = \begin{cases} t, & 0 \le t \le 2 \\ \sin(t) & t > 2 \end{cases}.$$

Problem 4 Calculate the Laplace transform of the following function:

$f(t) = te^{-2t} + t\sin(t)$

Problem 5 Compute the inverse Laplace transform of

$$F(s) = \frac{3s+9}{s^2 - 8s + 7}$$

Problem 6 Compute the inverse Laplace transform of

$$F(s) = \frac{e^{-2s}}{s(s^2 + 6s + 13)}$$

Problem 7 Compute the inverse Laplace transform of

$$F(s) = \frac{4s}{s^4 - 1}$$

Problem 8 Problem 33 from page 375 of Nagel Saff and Snider (6th ed.)

Problem 9 Problem 3 from page 383 of Nagel Saff and Snider (6th ed.)

Problem 10 Problem 7 from page 383 of Nagel Saff and Snider (6th ed.)

Problem 11 Problem 11 from page 383 of Nagel Saff and Snider (6th ed.)

Problem 12 Problem 29 from page 384 of Nagel Saff and Snider (6th ed.)

Problem 13 Problem 20 from page 396 of Nagel Saff and Snider (6th ed.)

Problem 14 Problem 35 from page 396 of Nagel Saff and Snider (6th ed.)

Problem 15 Problem 37 from page 397 of Nagel Saff and Snider (6th ed.)

Problem 16 Problem 10 from page 405 of Nagel Saff and Snider (6th ed.)

Problem 17 Problem 27 from page 406 of Nagel Saff and Snider (6th ed.)

Problem 18 Problem 13 from page 413 of Nagel Saff and Snider (6th ed.)

Problem 19 Problem 17 from page 413 of Nagel Saff and Snider (6th ed.)

Problem 20 Problem 25 from page 413 of Nagel Saff and Snider (6th ed.)

SOLUTION TO MISSION 8

PROBLEM 1 Calculate Laplace Transform of

$$f(t) = \sin t \cos 2t + \sin^2(3t)$$

Observe, by trigonometry, (*)

$$f(t) = \frac{1}{2}\sin(3t) - \frac{1}{2}\sin(t) + \frac{1}{2}\left(1 - \cos(6t)\right)$$

Thus

$$\boxed{\mathcal{L}\{f(t)\}(s) = \frac{1}{2}\left[\frac{3}{s^2+9} - \frac{1}{s^2+1} + \frac{1}{s} - \frac{s}{s^2+36}\right]}$$

Btw, we can derive (*) via imaginary exp. technique,

$$\sin(t)\cos(2t) = \frac{1}{2i}\left(e^{it} - e^{-it}\right)\frac{1}{2}\left(e^{2it} + e^{-2it}\right)$$

$$= \frac{1}{4i}\left(e^{3it} - e^{-3it} + e^{-it} - e^{it}\right)$$

$$= \frac{1}{2}\underbrace{\frac{1}{2i}\left(e^{3it} - e^{-3it}\right)}_{\sin(3t)} - \frac{1}{2}\underbrace{\frac{1}{2i}\left(e^{it} - e^{-it}\right)}_{\sin t}$$

$$\therefore \quad \underline{\sin(t)\cos(2t) = \frac{1}{2}\sin(3t) - \frac{1}{2}\sin(t)} \quad *$$

Problem 2

Calculate the Laplace transforms of $f(t) = e^t u(t-3) + \sin(t)u(t-6)$.

$$\mathcal{L}\{e^t u(t-3)\}(s) = e^{-3s}\mathcal{L}\{e^{t+3}\}(s)$$
$$= e^{-3s}\mathcal{L}\{e^3 e^t\}(s)$$
$$= e^{-3s}e^3\frac{1}{s-1}. \quad \text{I}$$

$$\mathcal{L}\{\sin t\, u(t-6)\}(s) = \mathcal{L}\{\sin(t+6)\}(s)\, e^{-6s}$$
$$= e^{-6s}\mathcal{L}\{\cos 6 \sin t + \sin 6 \cos t\}(s)$$
$$= e^{-6s}\left(\frac{\cos 6}{s^2+1} + \frac{(\sin 6)s}{s^2+1}\right) \quad \text{II}$$

Thus, collecting I & II,

$$\boxed{\mathcal{L}\{f(t)\}(s) = \frac{e^3 e^{-3s}}{s-1} + e^{-6s}\left(\frac{\cos 6 + s(\sin 6)}{s^2+1}\right)}$$

Problem 3 Calculate the Laplace transforms of the following function:

$$f(t) = \begin{cases} t, & 0 \le t \le 2 \\ \sin(t) & t > 2 \end{cases}. = t\,(u(t) - u(t-2)) + \sin t\; u(t-2)$$

$$f(t) = t\,u(t) + (\sin t - t)\,u(t-2)$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^2} + e^{-2s}\mathcal{L}\{\sin(t+2) - t - 2\}(s)$$

$$= \frac{1}{s^2} + e^{-2s}\mathcal{L}\{\sin(2)\cos t + \cos(2)\sin t - t - 2\}(s)$$

$$= \boxed{\frac{1}{s^2} + e^{-2s}\left[\frac{s(\sin(2)) + \cos(2)}{s^2+1} - \frac{1}{s^2} - \frac{2}{s}\right].}$$

Problem 4 Calculate the Laplace transforms of the following function:

$$f(t) = te^{-2t} + t\sin(t)$$

$$\mathcal{L}\{t(e^{-2t} + \sin t)\} = -\frac{d}{ds}\left[\mathcal{L}\{e^{-2t} + \sin t\}(s)\right]$$

$$= -\frac{d}{ds}\left[\frac{1}{s+2} + \frac{1}{s^2+1}\right]$$

$$= -\left[\frac{-1}{(s+2)^2} - \frac{2s}{(s^2+1)^2}\right]$$

$$= \boxed{\frac{1}{(s+2)^2} + \frac{2s}{(s^2+1)^2}.}$$

Problem 5 Compute the inverse Laplace transform of

$$F(s) = \frac{3s+9}{\underbrace{s^2-8s+7}_{(s-1)(s-7)}} = \frac{A}{s-1} + \frac{B}{s-7}$$

$$3s+9 = A(s-7) + B(s-1)$$

$\underline{s=7}$ $\quad 30 = 6B \quad \therefore \quad \underline{B=5}.$

$\underline{s=1}$ $\quad 12 = -6A \quad \therefore \quad \underline{A=-2}$

$$\mathcal{L}^{-1}\{F(s)\}(t) = \mathcal{L}^{-1}\left\{\frac{5}{s-7} - \frac{2}{s-1}\right\}(t)$$

$$= \boxed{5e^{7t} - 2e^{t}.}$$

Problem 6 Compute the inverse Laplace transform of

$$F(s) = \frac{e^{-2s}}{s(s^2+6s+13)} = e^{-2s}\, G(s)$$

this format anticipates the needed format for $\mathcal{L}^{-1}$ from sheet.

$$G(s) = \frac{1}{s((s+3)^2+4)} = \frac{A}{s} + \frac{B(s+3)+2C}{(s+3)^2+4}$$

$$1 = A(s^2+6s+13) + s(B(s+3)+2C)$$

$$1 = s^2(A+B) + s(6A+3B+2C) + 13A$$

Const | $1 = 13A \quad \therefore \quad \underline{A = 1/13}.$

s | $0 = 6A+3B+2C \Rightarrow B = \frac{-1}{3}\left(\frac{6}{13}+2C\right)$

s^2 | $0 = A+B \Rightarrow B = -A = \frac{-1}{13}$ easier, I'll stick with this. $\underline{B = -1/13}$.

Obviously, solve s-eqn for C,

$$C = -\frac{1}{2}(6A+3B) = \frac{-1}{2}\left(\frac{6}{13}-\frac{3}{13}\right) = \frac{-1}{2}\left(\frac{3}{13}\right) = \underline{\frac{-3}{26} = C}.$$

Thus,

$$G(s) = \frac{1}{13}\,\frac{1}{s} + \frac{\frac{-1}{13}(s+3) - \frac{3}{26}(2)}{(s+3)^2+2^2}$$

Hence,

$$g(t) = \mathcal{L}^{-1}\{G(s)\}(t) = \underline{\frac{1}{13} - \frac{1}{13}e^{-3t}\cos(2t) - \frac{3}{26}e^{-3t}\sin(2t)}.$$

Consequently,

$$\mathcal{L}^{-1}\{e^{-2s}G(s)\}(t) = g(t-2)\,u(t-2)$$

$$\Rightarrow \boxed{f(t) = \frac{1}{13}\left(1 - e^{-3(t-2)}\left[\cos(2(t-2)) + \frac{3}{2}\sin(2(t-2))\right]\right)u(t-3)}$$

Problem 7 Compute the inverse Laplace transform of

$$F(s) = \frac{4s}{s^4-1} = \frac{4s}{(s^2+1)(s^2-1)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2-1}$$

$$4s = (s^2-1)(As+B) + (s^2+1)(Cs+D)$$

$$\begin{aligned} s=1 \mid\ & 4 = 2(C+D) \\ s=-1 \mid\ & -4 = 2(-C+D) \end{aligned} \quad \begin{aligned} (+) &\Rightarrow 0 = 4D \ \therefore\ \underline{D=0}. \\ (-) &\Rightarrow 8 = 4C \ \therefore\ \underline{C=2}. \end{aligned}$$

$$\begin{aligned} s=i \mid\ & 4i = -2(iA+B) \\ s=-i \mid\ & -4i = -2(-iA+B) \end{aligned} \quad \begin{aligned} (+) &\Rightarrow 0 = -4B \ \therefore\ B=0 \\ (-) &\Rightarrow 8i = -4iA \ \therefore\ \underline{A=-2}. \end{aligned}$$

$$F(s) = \frac{-2s}{s^2+1} + \frac{2s}{s^2-1}$$

Thus,

$$\boxed{\mathcal{L}^{-1}\{F(s)\}(t) = -2\cos(t) + 2\cosh(t)}$$

Note, you might have gone the exponential road:

$$\frac{2s}{s^2-1} = \frac{1}{s-1} + \frac{1}{s+1} \xrightarrow{\mathcal{L}^{-1}} e^{t} - e^{-t} = 2\cosh t$$

either method obtains same answer.

$\boxed{P10}$ #7 of p. 383

$$y'' - 7y' + 10y = 9\cos t + 7\sin t, \quad y(0) = 5, \; y'(0) = -4$$

$$s^2 Y - 5s + 4 - 7(sY - 5) + 10Y = \frac{9s+7}{s^2+1}$$

$$(s^2 - 7s + 10)Y = 5s - 39 + \frac{9s+7}{s^2+1}$$

$$Y = \frac{5s-39}{s^2-7s+10} + \frac{9s+7}{(s^2+1)(s^2-7s+10)}$$

C.A.S.

$$= \frac{29}{3(s-2)} - \frac{14}{3(s-5)} + \frac{s}{s^2+1} + \frac{2}{3(s-5)} - \frac{5}{3(s-2)}$$

$$= \frac{s}{s^2+1} - \frac{4}{s-5} + \frac{8}{s-2}$$

In short,

$$\frac{5s-39}{(s-2)(s-5)} + \frac{9s+7}{(s^2+1)(s-2)(s-5)} = \frac{As+B}{s^2+1} + \frac{C}{s-5} + \frac{D}{s-2}$$

do some algebra to find $A = 1$, $B = 0$, $C = -4$, $D = 8$.

Consequently,

$$\mathcal{L}^{-1}\{Y\}(t) = \boxed{y(t) = \cos(t) - 4e^{5t} + 8e^{2t}}$$

P11 #11 of p. 383 Solve via Laplace,

$$\boxed{y'' - y = t - 2, \quad y(2) = 3, \quad y'(2) = 0}$$

Let $w(t) = y(t+2)$ hence $w(0) = y(2) = 3$, $w'(0) = y'(2) = 0$

Note that $y''(t+2) = w''(t)$ and $t - 2 \mapsto (t+2) - 2 = t$ if we replace t with $t+2$,

$$\left(y''(t) - y(t) = t - 2\right) \xrightarrow{\;t\;} y''(t+2) - y(t+2) = t$$

Thus $w'' - w = t$ with $w(0) = 3$ and $w'(0) = 0$

$$s^2 W - 3s - W = \frac{1}{s^2}$$

$$(s^2 - 1)W = 3s + \frac{1}{s^2}$$

$$W = \frac{3s}{s^2 - 1} + \frac{1}{s^2(s^2-1)} = \frac{3s^3 + 1}{s^2(s^2-1)}$$

$$W = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{D}{s+1} = \frac{3s^3+1}{s^2(s^2-1)}$$

$$As(s^2-1) + B(s^2-1) + Cs^2(s+1) + Ds^2(s-1) = 3s^3 + 1$$

$s = 0$: $-B = 1$ $\therefore$ $B = -1$.

$s = 1$: $2C = 4$ $\therefore$ $C = 2$

$s = -1$: $-2D = -3 + 1 = -2$ $\therefore$ $D = 1$.

s^3: $A + C + D = 3 \Rightarrow A = 3 - C - D = 3 - 2 - 1 = 0$.

Thus $$W = \frac{-1}{s^2} + \frac{2}{s-1} + \frac{1}{s+1}$$

$\therefore$ $w(t) = -t + 2e^{t} + e^{-t}$

$$\Rightarrow y(t) = w(t-2) = -(t-2) + 2e^{t-2} + e^{-(t-2)}$$

$$\therefore \boxed{y(t) = 2 - t + 2e^{t-2} + e^{2-t}}$$

P12 # 29 from p. 384

$$y'' - 4y' + 3y = 0 \quad \text{with} \quad y(0) = a, \; y'(0) = b$$

$$s^2 Y - as - b - 4(sY - a) + 3Y = 0$$

$$(s^2 - 4s + 3) Y = as + b - 4a$$

$$Y = \frac{as + b - 4a}{s^2 - 4s + 3}$$

$$\frac{as + b - 4a}{s^2 - 4s + 3} = \frac{C}{s-1} + \frac{D}{s-3}$$

$$as + b - 4a = C(s-3) + D(s-1) = (C+D)s - 3C - D$$

We find,

$$C + D = a$$
$$-3C - D = b - 4a$$

$$-2C = b - 3a \quad \therefore \quad \underline{C = \frac{3a-b}{2}}.$$

$$\Rightarrow D = a - C = a - \left(\frac{3a-b}{2}\right) = \underline{\frac{b-a}{2} = D}.$$

Thus, $Y = \left(\frac{3a-b}{2}\right)\left(\frac{1}{s-1}\right) + \left(\frac{b-a}{2}\right)\left(\frac{1}{s-3}\right)$

$$\therefore \boxed{y(t) = \left(\frac{3a-b}{2}\right)e^{t} + \left(\frac{b-a}{2}\right)e^{3t}}$$

$\boxed{P13}$ #20 pg. 296

Solve $\quad I'' + 4I = \begin{cases} 3\sin t & 0 < t < 2\pi \\ 0 & t > 2\pi \end{cases} = 3\sin t\,[u(t) - u(t-2\pi)]$

subject $\quad I(0) = 1$ and $I'(0) = 3$

$$s^2\mathcal{I} - s - 3 + 4\mathcal{I} = 3\mathcal{L}\{\sin t\, u(t) - \sin t\, u(t-2\pi)\}(s)$$

$$= \frac{3}{s^2+1} - 3e^{-2\pi s}\mathcal{L}\{\sin(t+2\pi)\}(s)$$

$$= \frac{3}{s^2+1} - 3e^{-2\pi s}\left(\frac{1}{s^2+1}\right)$$

Hence,

$$(s^2+4)\mathcal{I} = s + 3 + \frac{3}{s^2+1} - e^{-2\pi s}\left(\frac{3}{s^2+1}\right)$$

$$\mathcal{I} = \frac{s+3}{s^2+4} + \frac{3}{(s^2+4)(s^2+1)} - \frac{3e^{-2\pi s}}{(s^2+4)(s^2+1)}$$

$$\frac{3}{(s^2+4)(s^2+1)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+1}$$

$$3 = (As+B)(s^2+1) + (Cs+D)(s^2+4)$$

$$3 = s^3(A+C) + s^2(B+D) + s(A+4C) + B + 4D$$

Thus,

$A + C = 0 \quad \longrightarrow \quad \underline{C = 0}. \Rightarrow \underline{A = 0}.$

$B + D = 0 \quad \longrightarrow \quad D = -B$

$A + 4C = 0 \qquad\qquad\qquad B - 4B = 3$

$B + 4D = 3 \qquad\qquad\qquad -3B = 3 \;\therefore\; \underline{B = -1}$

and $\underline{D = 1}$.

Therefore,

$$\mathcal{I} = \frac{s+3}{s^2+4} + \left(\frac{3}{s^2+1} - \frac{3}{s^2+4}\right)\left(1 - e^{-2\pi s}\right)$$

$$\Rightarrow I(t) = \cos(2t) + \frac{3}{2}\sin(2t) + 3\sin t - 3\sin(2t) + 2$$

$$- \left[3\sin(t-2\pi) - 3\sin(2(t-2\pi))\right]u(t-2\pi)$$

$$\therefore \boxed{I(t) = \cos(2t) - \frac{3}{2}\sin(2t) + 3\sin t - [3\sin t - 3\sin(2t)]u(t-2\pi)}$$

$\boxed{P14}$ #35 p. 396

$$z'' + 3z' + 2z = e^{-3t}u(t-2), \quad z(0)=2, \; z'(0) = -3$$

$$s^2 Z - 2s + 3 + 3(sZ - 2) + 2Z = \mathcal{L}\{e^{-3(t+2)}\}(s)\, e^{-2s}$$

$$(s^2+3s+2)Z - 2s - 3 = e^{-6}\left(\frac{1}{s-3}\right)e^{-2s}$$

$$Z = \frac{2s+3}{s^2+3s+2} + \frac{e^{-6}}{(s-3)(s^2+3s+2)}e^{-2s}$$

$$Z = \frac{1}{s+2} + \frac{1}{s+1} + e^{-6}\left(\frac{1}{20(s-3)} + \frac{1}{5(s+2)} - \frac{1}{4(s+1)}\right)e^{-2s}$$

$$\Rightarrow \boxed{z(t) = e^{-2t} + e^{-t} + e^{-6}\left(\frac{1}{20}e^{3(t-2)} + \frac{1}{5}e^{-2(t-2)} - \frac{1}{4}e^{-t+2}\right)u(t-2)}$$

(I differ with the text's answer key here... there may be an error...)

$\boxed{P15}$ #37 p. 397 where $y(0)=1$ and $y'(0)=3$, solve

$$y'' + 4y = \begin{cases} \sin t & 0 \le t \le 2\pi \\ 0 & t > 2\pi \end{cases} = \sin t\,[u(t) - u(t-2\pi)]$$

$$s^2 Y - s - 3 + 4Y = \frac{1}{s^2+1} - e^{-2\pi s}\frac{1}{s^2+1}$$ (like #20 of p. 396)

$$Y = \frac{s+3}{s^2+4} + \frac{1}{(s^2+1)(s^2+4)}\left(1 - e^{-2\pi s}\right)$$

$$Y = \frac{s+3}{s^2+4} + \frac{1}{3}\left(\frac{1}{s^2+1} - \frac{1}{s^2+4}\right)\left(1 - e^{-2\pi s}\right)$$

$u(t-2\pi)$

$$\therefore\; y(t) = \cos(2t) + \frac{3}{2}\sin(2t) + \frac{1}{3}\left(\sin t - \frac{1}{2}\sin(2t)\right) - \frac{1}{3}\left(\sin(t+2\pi) - \frac{1}{2}\sin(2(t+2\pi))\right)$$

$$\therefore\; \boxed{y(t) = \cos(2t) + \sin(2t)\cdot\frac{4}{3} + \frac{1}{3}\sin t - \frac{1}{3}\left(\sin t - \frac{1}{2}\sin(2t)\right)u(t-2\pi)}$$

P16 #10 p. 405 | Calculate the (Laplace Transform)$^{-1}$ via convolution,

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3(s^2+1)}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}(t) * \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}(t)$$

$$= \left(\frac{1}{2}t^2\right) * \left(\sin(t)\right)$$

$$= \int_0^t \frac{1}{2}(t-v)^2 \sin(v)\,dv$$

$$= \int_0^t \frac{1}{2}\left(t^2 - 2tv + v^2\right)\sin(v)\,dv$$

$$= \frac{1}{2}\int_0^t t^2 \sin(v)\,dv - t\int_0^t v\sin(v)\,dv + \frac{1}{2}\int_0^t v^2 \sin(v)\,dv$$

$$= \frac{t^2}{2}\left(-\cos(v)\Big|_0^t\right) - t\left(\sin(v) - v\cos(v)\right)\Big|_0^t + \frac{1}{2}\left(2v\sin v - v^2\cos v + 2\cos v\right)\Big|_0^t$$

$$= \frac{t^2}{2}\left(1-\cos t\right) - t\left(\sin t - t\cos t\right) + \frac{1}{2}\left(2t\sin t - t^2\cos t + 2\cos t\right) - \frac{1}{2}\left(2\cos(0)\right)$$

$$= \cancel{-t^2\cos t} + \cancel{t^2\cos t} + \frac{t^2}{2} - \cancel{t\sin t} + \cancel{t\sin t} + \cos t - 1$$

$$= \boxed{\frac{1}{2}t^2 + \cos t - 1}$$

Check my answer,

$$\mathcal{L}\left\{\frac{1}{2}t^2 + \cos t - 1\right\} = \frac{1}{s^3} + \frac{s}{s^2+1} - \frac{1}{s}$$

$$= \frac{s^2+1 + s^4 - (s^2+1)s^2}{s^3(s^2+1)}$$

$$= \frac{1}{s^3(s^2+1)} \qquad \text{phew.}$$

P17 # 27 from p. 406

$$\boxed{y'' - 2y' + 5y = g(t), \quad y(0) = 0, \quad y'(0) = 2.}$$

$$s^2 Y - s(0) - 2 - 2(sY - 0) + 5Y = G = \mathcal{L}\{g\}$$

$$(s^2 - 2s + 5)Y = 2 + G$$

$$Y = \frac{2}{s^2 - 2s + 5} + \frac{1}{s^2 - 2s + 5} G$$

$$\underbrace{H(s) = \frac{1}{s^2 - 2s + 5} = \frac{1}{(s-1)^2 + 4}}_{\text{transfer function}} \Rightarrow \underbrace{h(t) = \frac{e^t \sin(2t)}{2}}_{\text{impulse responce fnct.}}$$

$$Y = H(s) G(s) + \frac{2}{(s-1)^2 + 4}$$ and by convolution

$$\mathcal{L}^{-1}\{HG\} = (h * g)(t)$$

$$\Rightarrow \boxed{y(t) = e^t \sin(2t) + \int_0^t \frac{1}{2} e^{t-v} \sin(2(t-v))\, g(v)\, dv}$$

$\boxed{P18}$ #13 p. 413

$$w'' + w = \delta(t-\pi), \quad w(0) = 0, \quad w'(0) = 0$$

$$s^2 W + W = e^{-\pi s}$$

$$W = \left(\frac{1}{s^2+1}\right) e^{-\pi s}$$

$$\Rightarrow \boxed{w(t) = \sin(t-\pi)\, u(t-\pi)}$$

$\boxed{P19}$ # 17 p. 413

$$y'' - y = 4\delta(t-2) + t^2, \quad y(0) = 0, \quad y'(0) = 3$$

$$s^2 Y - 3 - Y = 4e^{-2s} + \frac{2}{s^3}$$

$$Y = \frac{3}{s^2-1} + \frac{2}{s^3(s^2-1)} + \frac{4}{s^2-1} e^{-2s}$$

$$Y = \frac{3}{s^2-1} - \frac{2}{s^3} - \frac{2}{s} + \frac{1}{s+1} + \frac{1}{s-1} + \frac{4}{s^2-1} e^{-2s}$$

$$\boxed{y(t) = 3\sinh(t) - 2t^2 - 2 + e^{-t} + e^{t} + 4\sinh(t-2)\, u(t-2)}$$

(see text for alt. formulation in terms of exponentials)

$\boxed{P20}$ # 25 p. 413

$$y'' + 4y' + 8y = \delta(t), \quad y(0) = y'(0) = 0$$

$$s^2 Y + 4sY + 8Y = 1$$

$$Y = \frac{1}{s^2+4s+8} = \frac{1}{(s+2)^2+4}$$

$$\boxed{y(t) = \frac{1}{2} e^{-2t} \sin(2t)}$$