PHYSICS 231	MISSION 1: VECTORS, PATHS, ONE-DIMENSIONAL MOTION (20+4)PTS

Your solutions should be neat, correct and complete. Full credit is not necessarily attained from the correct answer, you can lose points if the solution is not readable. Your solution should be readable to someone who has not read the problem statement. If I have to think about what your calculation means then it's not complete. Numerical answers must be given in scientific notation for credit to be awarded. Missing or incorrect units on your answer automatically deducts $\frac{1}{2}$ of the credit. Finally, the answer must be boxed when there is a particular answer to find. Derivation problems are exceptions to this rule.

Recommended Homework from Textbook (Serway):
Chapter 2 #'s 9, 11, 29, 39 & Chapter 3 #'s 15, 17, 35, 37.

Recommended Homework from Recommended Textbook (Young & Freedman, 9th ed):
Chapter 1 (vectors and conversion) #'s 27, 31, 35, 37, 41, 51, 60, 67
Chapter 2 (1D-motion) #'s 9, 15, 17, 19, 23, 27, 35, 37, 41, 45, 47, 51, 55, 57, 61, 65, 67, 71, 73, 75, 77

Suggested Reading You may find the following helpful resources beyond lecture,

(a.) Lectures 2 and 5 as posted on the course website,

(b.) Chapters 1, 2 and 3 of the required text.

Problem 1: (2pts) Let $\vec{\mathbf{A}} = \langle 3, 4\rangle$ and $\vec{\mathbf{B}} = \langle 12, 9\rangle$. Find A, B, $\widehat{\mathbf{A}}$, $\widehat{\mathbf{B}}$ and $\angle(\vec{A}, \vec{B})$.

$$A = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{9+16} = \sqrt{25} = \boxed{5}$$

$$B = \sqrt{\vec{B}\cdot\vec{B}} = \sqrt{144+81} = \sqrt{225} = \boxed{15}$$

$$\hat{A} = \frac{1}{A}\vec{A} = \boxed{\langle 3/5, 4/5\rangle}$$

$$\hat{B} = \frac{1}{B}\vec{B} = \boxed{\langle 12/15, 9/15\rangle}$$

$$\hat{A}\cdot\hat{B} = \cos\theta = \left(\frac{3}{5}\right)\left(\frac{12}{15}\right) + \left(\frac{4}{5}\right)\left(\frac{9}{15}\right) = 24/25 = 0.96$$

$$\theta = \cos^{-1}(0.96)$$

$$\boxed{\angle(\vec{A}, \vec{B}) = 16.26^\circ}$$

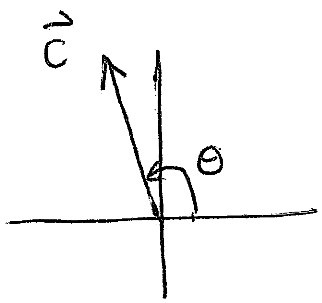
Notice $\vec{A}\cdot\vec{B} = AB\cos\theta \Rightarrow \left(\frac{\vec{A}}{A}\right)\cdot\left(\frac{\vec{B}}{B}\right) = \cos\theta$

$\therefore\ \hat{A}\cdot\hat{B} = \cos\theta$

Problem 2: (2pts)Let $\vec{A}$ be a vector of length 10 with standard angle $\theta = 110^o$. Let $\vec{B} = 4\hat{\mathrm{x}} - 10\hat{\mathrm{y}}$. Find the magnitude and direction of $\vec{C} = 2\vec{A} - \vec{B}$.

$$\vec{A} = A\langle \cos\theta, \sin\theta \rangle = \langle 10\cos 110, 10\sin 110 \rangle = \langle -3.42, 9.40 \rangle$$

$$\vec{C} = 2\vec{A} - \vec{B} = 2\langle -3.42, 9.40\rangle - \langle 4, -10\rangle$$
$$= \langle 2(-3.42) - 4, 2(9.40) + 10 \rangle$$
$$= \langle -10.84, 28.8 \rangle$$



$$C = \sqrt{\vec{C}\cdot\vec{C}} \cong \boxed{30.8} \quad \& \quad \boxed{\theta = 110.63^\circ}$$

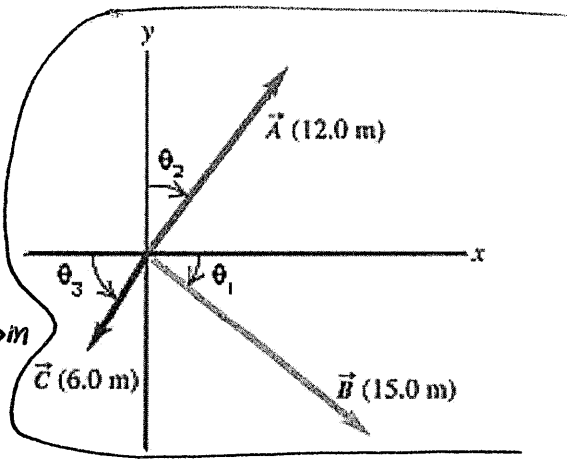
$$\theta = 90^\circ + \tan^{-1}\left(\frac{10.84}{28.8}\right) = 90^\circ + 20.63^\circ$$

Alternatively, the direction is $\boxed{\hat{C} \cong \langle 0.35, 0.94 \rangle}$

Problem 3: (2pts) You are given the angles in the diagram below are $\theta_1 = 44^o$, $\theta_2 = 30^o$ and $\theta_3 = 59^o$

(a.) find $A_1, A_2, B_1, B_2, C_1, C_2$ for which $\vec{A} = \langle A_1, A_2\rangle$, and $\vec{B} = \langle B_1, B_2\rangle$ and $\vec{C} = \langle C_1, C_2\rangle$.

(b.) Suppose $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. Find a vector in the direction of $\vec{D}$ of length $20\,m$.



(a.) $\vec{A} = \langle 12m \sin 30^\circ, 12m\cos 30^\circ \rangle = \langle 6, 10.39 \rangle m$

$\vec{B} = \langle 15m\cos 44^\circ, -15m\sin 44^\circ \rangle = \langle 10.79, -10.42\rangle m$

$\vec{C} = \langle -6m\cos 59^\circ, -6m\sin 59^\circ \rangle = \langle -3.09, -5.14 \rangle m$

(b.) $\vec{D} = \vec{A} + \vec{B} + \vec{C}$

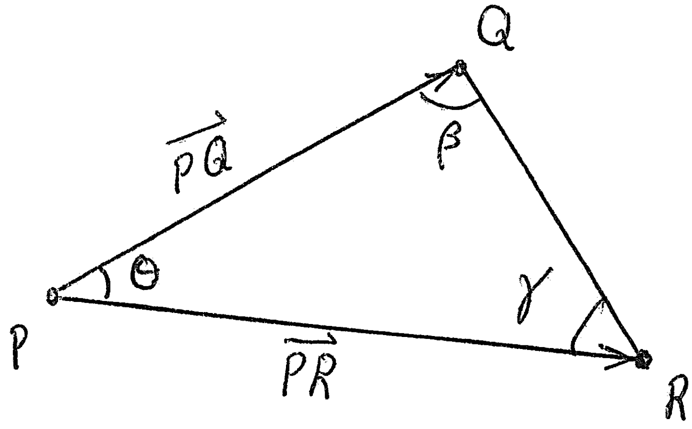
$= \langle 6 + 10.79 - 3.09, 10.39 - 10.42 - 5.14 \rangle m$

$= \langle 13.7m, -5.17m \rangle \Rightarrow D = \sqrt{\vec{D}\cdot\vec{D}} = \sqrt{214.4m^2} \cong 14.64\,m$

$\hat{D} = \frac{1}{14.64m}\langle 13.7m, -5.17m \rangle = \langle 0.936, -0.353 \rangle \cong \langle 0.94, -0.35 \rangle$

$$\underbrace{20\hat{D}}_{\text{vector length 20 in direction of } \vec{D}} = 20\langle 0.94, -0.35 \rangle \cong \boxed{\langle 18.8, -7 \rangle}$$

Problem 4: (2pts) A triangle PQR is formed by the triple of points $P = (1,1,1)$ and $Q = (0,2,0)$ and $R = (3,2,7)$. Find the lengths and angles of this triangle. Present your answer as a picture with the sides and angles labeled neatly (you do not need to draw it to scale or perspective, please use vectors to solve this problem)



$\vec{PQ} = Q - P = \langle -1, 1, -1 \rangle$

$\vec{PR} = R - P = \langle 2, 1, 6 \rangle$

$\vec{QP} = P - Q = \langle 1, -1, 1 \rangle$

$\vec{QR} = R - Q = \langle 3, 0, 7 \rangle$

① $\vec{PQ} \cdot \vec{PR} = \|\vec{PQ}\| \cdot \|\vec{PR}\| \cos\theta$

② $\vec{QP} \cdot \vec{QR} = \|\vec{QP}\| \cdot \|\vec{QR}\| \cos\beta$

Yields,

① $-2 + 1 - 6 = \sqrt{3}\sqrt{41} \cos\theta \Rightarrow \cos\theta = \dfrac{-7}{\sqrt{3}\sqrt{41}}$

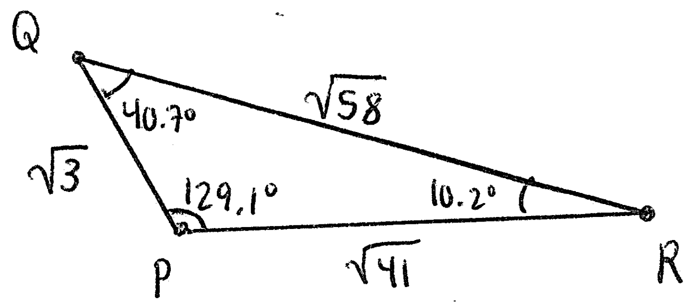
$\therefore \theta = \cos^{-1}\left(\dfrac{-7}{\sqrt{3}\sqrt{41}}\right) = \boxed{129.1^\circ}$

② $3 - 0 + 7 = \sqrt{3}\sqrt{58} \cos\beta$

$\beta = \cos^{-1}\left(\dfrac{10}{\sqrt{3}\sqrt{58}}\right) = \boxed{40.7^\circ}$

Then $\theta + \beta + \gamma = 180^\circ \Rightarrow \gamma = 180^\circ - 129.1^\circ - 40.7^\circ$

$\gamma = \boxed{10.2^\circ}$



$\sqrt{3} = 1.732...$

$\sqrt{41} = 6.403...$

$\sqrt{58} = 7.616...$

Problem 5: (2pts) A mouse travels the edge of a $12ft$ by $12ft$ room with corners we shall label A, B, C, D for convenience. Beginning at corner A and travelling to the diagonally opposite corner C. Then the mouse climbs some curtains near a shelf which is placed in the corner C some $6ft$ off the floor and jumps to the corner shelf to get some cheese which sitting on the shelf.

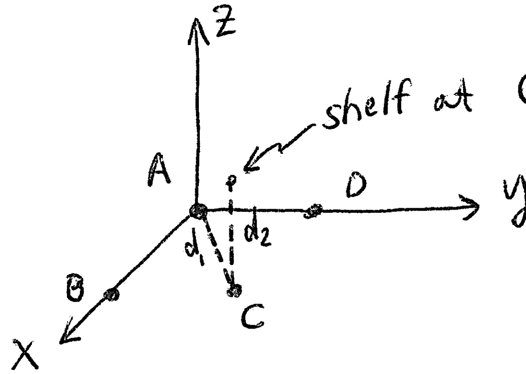
(a.) what distance to the mouse travel ?

(b.) using A as the origin $(0, 0, 0)$, what is the displacement of the mouse ?
(for specificity, let us place C at $(12ft, 12ft, 0)$)

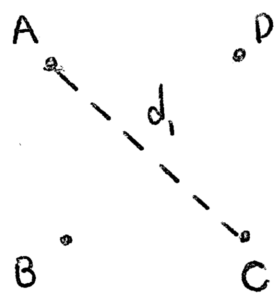
(c.) a hamster assassin is on top of a doll house at corner B and his position is $(12ft, 0, 2ft)$. A what angle about the horizontal does the hamster assassin need to aim his laser gun to shoot the mouse ?

"above"

shelf at $C + (6ft)\,\hat{z} = (12ft, 12ft, 6ft)$



(a.)



distance on floor

distance up curtains

$$\text{distance travel} = d_1 + d_2$$

16.97

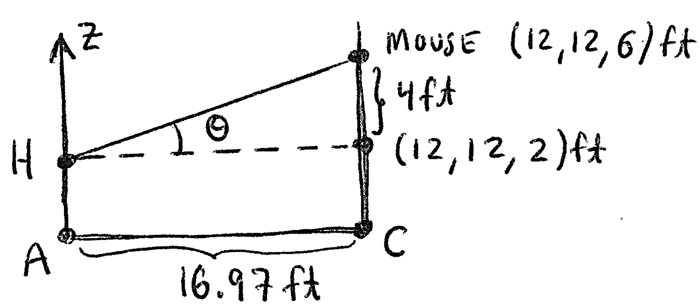
$$= \left(\sqrt{12^2 + 12^2} + 6\right) ft$$

$$= \boxed{22.97\ ft}$$

(b.) $\vec{d_1} + \vec{d_2} = \boxed{\langle 12, 12, 6 \rangle ft}$

Remark: you can either write ft inside or outside the vector, but it needs to be somewhere.

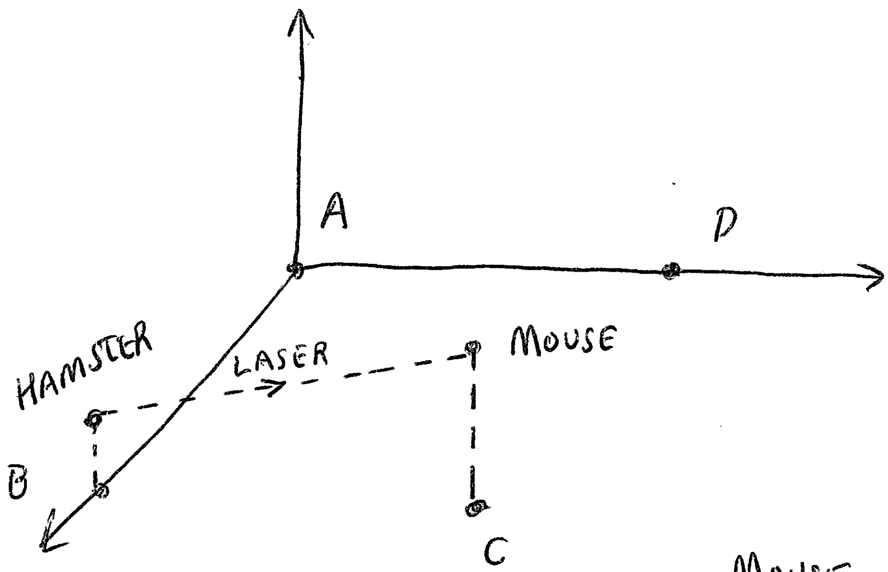
(c.)

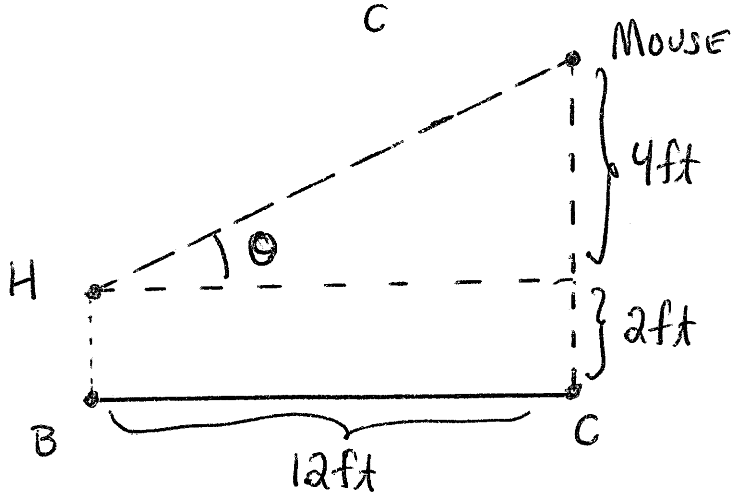


$$\theta = \tan^{-1}\left(\frac{4}{16.97}\right) \cong \boxed{13.26^\circ}$$

OOPS! I MISPLACED THE HAMSTER ASSASSIN. I'll DO IT OVER

PROBLEM 5 part C





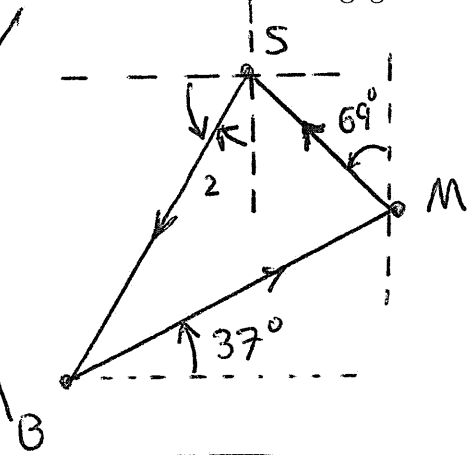
$$\theta = \tan^{-1}\left(\frac{4}{12}\right) = \boxed{18.43^\circ}$$

Problem 6: (2pts) A ferry transports tourists between three islands. It sails from the Island of Brandon to the Island of Magaga, 4.76 km away, in a direction 37.0° north of east. It then sails from the Island of Magaga to the Isle of Swanson in a direction in a direction 69° west of north. Finally it returns to the Island of Brandon sailing in a direction of 28° east of south.

(a.) Calculate the distance between Brandon and ~~Magaga~~ Swanson, (5.99 km)

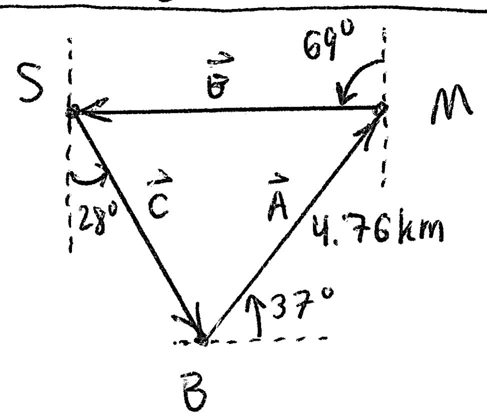
(b.) Calculate the distance between Magaga and Swanson. (6.99 km)

I leave this to show true process ☺



← impossible... So... I try again 2

Solution:



$\vec{A}$ = displacement from B → M

$\vec{B}$ = displacement from M → S

$\vec{C}$ = displacement from S → B

We know $A = 4.76\,km$ and we can geometrically determine $\hat{A}, \hat{B}, \hat{C}$ from the given angles, $\theta_{\vec{A}} = 37°$, $\theta_{\vec{B}} = 159°$, $\theta_{\vec{C}} = -62°$

Observe $\vec{A} + \vec{B} + \vec{C} = 0$

$$0 = A\langle \cos 37, \sin 37\rangle + B\langle \cos 159, \sin 159\rangle + C\langle \cos -62, \sin(-62)\rangle$$

$$0.8A - 0.93B + 0.47C = 0$$
$$0.6A + 0.36B - 0.88C = 0$$

$$\begin{bmatrix} -0.93B + 0.47C = -3.68km \\ 0.36B - 0.88C = -2.76km \end{bmatrix}$$

→ Solve to obtain,

$$\boxed{\begin{aligned} B &= 6.99\,km \\ C &= 5.99\,km \end{aligned}}$$

Problem 7: (2pts) Suppose we have three vectors $\vec{A}, \vec{B}, \vec{C}$ all of which are perpendicular to one another; $\angle(\vec{A}, \vec{B}) = 90^o$, $\angle(\vec{A}, \vec{C}) = 90^o$, $\angle(\vec{B}, \vec{C}) = 90^o$. Let $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. Prove that:

$$D^2 = A^2 + B^2 + C^2$$

where A, B, C denote the magnitudes of $\vec{A}, \vec{B}, \vec{C}$ respectively. Hint: do **not** attempt a picture. Instead, use $D^2 = \vec{D} \bullet \vec{D}$ and the properties of the dot-product to formulate your solution.

Note, $\vec{A} \cdot \vec{B} = 0$, $\vec{A} \cdot \vec{C} = 0$ and $\vec{B} \cdot \vec{C} = 0$ since $\cos 90^0 = 0$

$$D^2 = \vec{D} \cdot \vec{D}$$

$$= (\vec{A} + \vec{B} + \vec{C}) \cdot (\vec{A} + \vec{B} + \vec{C})$$

$$= \vec{A} \cdot \vec{A} + \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C} + \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B} + \vec{B} \cdot \vec{C} + \vec{C} \cdot \vec{A} + \vec{C} \cdot \vec{B} + \vec{C} \cdot \vec{C}$$

$$= A^2 + B^2 + C^2$$

as

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = AB \cos 90^0 = 0$$

$$\vec{A} \cdot \vec{C} = \vec{C} \cdot \vec{A} = AC \cos 90^0 = 0$$

$$\vec{B} \cdot \vec{C} = \vec{C} \cdot \vec{B} = BC \cos 90^0 = 0 \,.//$$

Problem 8: (2pts) Suppose $\vec{r}(t) = \langle \alpha t, \beta\cos(kt), \beta\sin(kt)\rangle$ gives the position at time t for a particle where α, β, k are constants.

(a.) calculate the velocity at time t,

(b.) calculate the acceleration at time t,

(c.) $\alpha = 2\,m/s$ and $\beta = 3\,m$ and $k = 2\pi/s$ find the displacement of the particle from $t = 0$ to $t = 3\,s$.

(d.) Calculate the distance travelled from $t = 0$ to $t = 3\,s$ (may use numerical integrator for integration here if you wish)

(a.) $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}\langle \alpha t, \beta\cos kt, \beta\sin kt\rangle$

$$\boxed{\vec{v}(t) = \langle \alpha, -\beta k\sin kt, \beta k\cos kt\rangle}$$

(b.) $\boxed{\vec{a}(t) = \langle 0, -\beta k^2\cos kt, -\beta k^2\sin kt\rangle}$

(c.) $\Delta\vec{r} = \vec{r}(3s) - \vec{r}(0)$ $\quad \alpha = 2\frac{m}{s}, \ \beta = 3m, \ k = \frac{2\pi}{s}$

$= \langle 6m, 3m\cos(6\pi), 3m\sin(6\pi)\rangle - \langle 0, 3m, 0\rangle$

$= \boxed{\langle 6m, 0, 0\rangle}$

(d.) $v^2 = \|\vec{v}\|^2 = \alpha^2 + \beta^2 k^2$ since $\sin^2\theta + \cos^2\theta = 1$

Speed $= \sqrt{4 + 9(4\pi^2)}\,\frac{m}{s} \cong 18.96\,\frac{m}{s}$

$\Delta s = \displaystyle\int_0^{3s} v\,dt \cong (18.96\,\tfrac{m}{s})(3s) \cong \boxed{56.87\,m}$

(no need for numerical $\int$ as it happens, but if you didn't see $\cos^2\theta + \sin^2\theta = 1$ then that makes this way harder)

Problem 9: (3pts) Consider the graph of velocity verse time given below.

(a.) What is the displacement of the particle over the time interval $-3\,s \le t \le 3\,s$?

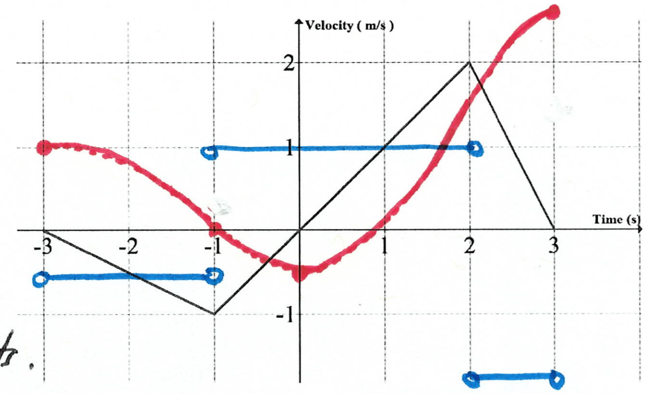
$$\underbrace{\text{area} = \tfrac{1}{2}(\text{base})(\text{height})}_{\text{triangle}}$$

$$\Delta x = \int_{-3s}^{3s} v\,dt = \left(-\tfrac{1}{2}(3)(1) + \tfrac{1}{2}(3)(2)\right)m = \boxed{1.5m}$$

(b.) What is the distance travelled over the time interval $-3\,s \le t \le 3\,s$?

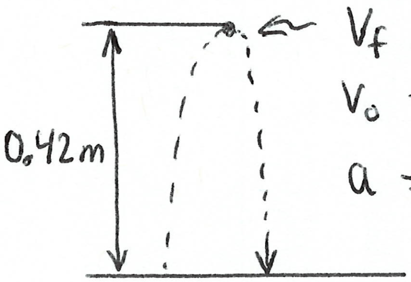
$$\Delta S = \int_{-3s}^{3s} |v|\,dt = \left(\tfrac{3}{2} + \tfrac{6}{2}\right)m = \boxed{4.5m}$$

(c.) Given $x = 1.0\,m$ when $t = -3\,s$. Graph the position and acceleration on the given graph.



- max/min for x happens where $v = 0$.
- jumps in v are at inflection pts. for this example.

Problem 10: (1pts) If a flea can jump straight to a height of $0.420\,m$, what is it initial speed as it leaves the ground ? Also, how much time is it in the air ?



$V_f = 0$

$V_o = ?$

$a = -g$

$$V_f^2 = V_o^2 - 2g\Delta y$$

$$V_o = \sqrt{2g\Delta y}$$

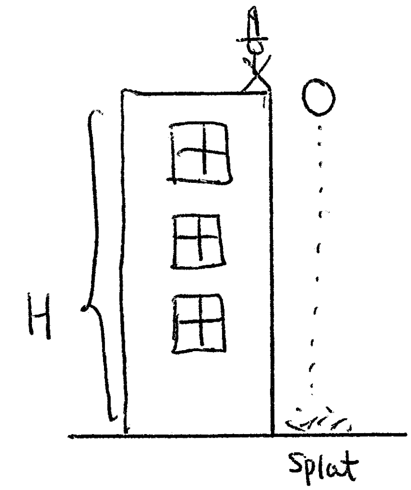
$$= \sqrt{2(9.8\,m/s^2)(0.42m)}$$

$$= \boxed{2.87\,m/s}$$

$$V_f = V_o - gt$$

$$t = \frac{V_o - V_f}{g} = \frac{2.87\,m/s}{9.8\,m/s^2} = \boxed{0.29s}$$

Problem 11: (2pts) A physics student with too much free time drops a watermelon from the ~~rood~~ roof of a building. He hears the sound of the watermelon going "splat" 2.34 s later. How high is the building ? Assume the speed of sound is 340 m/s in the day in question.



t_1 = time to drop

t_2 = time for sound to travel back

$$t_1 + t_2 = 2.34s$$

$$\underbrace{H = (340\,\tfrac{m}{s})t_2}_{\text{constant velocity motion for sound}} \quad \& \quad \underbrace{y = H - \tfrac{1}{2}gt^2}$$

$$0 = H - \tfrac{1}{2}gt_1^2$$

$$H = \tfrac{1}{2}gt_1^2$$

$$(340\,\tfrac{m}{s})t_2 = \tfrac{1}{2}gt_1^2 \Rightarrow t_2 = 0.01441 t_1^2 \; *$$

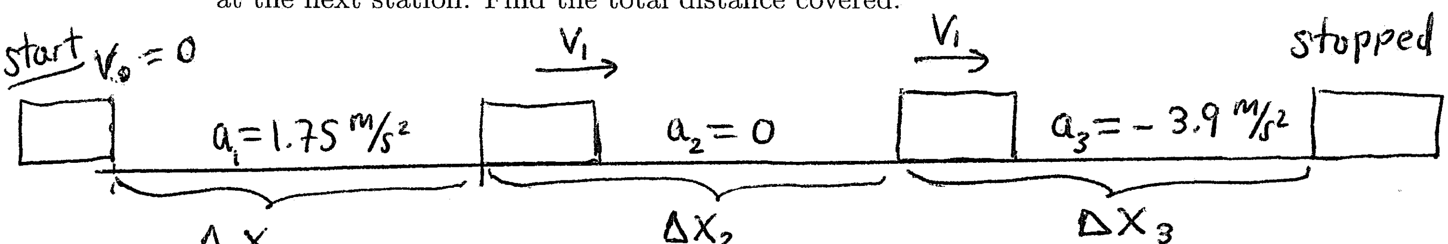
$$\text{By } * \quad t_1 + t_2 = 0.01441 t_1^2 + t_1 = 2.34 \rightarrow t_1 = 2.26 \text{ or } -71.7$$

extraneous

$$H = \tfrac{1}{2}gt_1^2$$

$$\boxed{H = 25.03\,m}$$

Problem 12: (2pts) A subway train starts from rest at a station and accelerates at $1.75 m/s^2$ for 12.0 s. It runs at constant speed for 75.0 s and slows down at a rate of $3.90 m/s^2$ until it stops at the next station. Find the total distance covered.



$$\Delta X_2 = V_1 \Delta t_2 \qquad V_f^2 = V_1^2 + 2a_3 \Delta X_3$$

$$\Delta t_2 = 75.0s \qquad V_f = 0$$

$$\Delta X_1 = \tfrac{1}{2}a_1(\Delta t_1)^2 \qquad \Delta X_3 = \frac{-V_1^2}{2a_3}$$

$$\Delta X_1 = \tfrac{1}{2}(1.75\,\tfrac{m}{s^2})(12.0s)^2 = 126m \qquad \Delta X_3 = \frac{-(21\,m/s)^2}{2(-3.9\,m/s^2)}$$

$$V_1 = a_1 \Delta t_1 = (1.75\,\tfrac{m}{s^2})(12.0s) = 21\,\tfrac{m}{s} \qquad \Delta X_3 = 56.5m$$

$$\Delta X_2 = V_1 \Delta t_2 = (21\,\tfrac{m}{s})(75s) = 1575m$$

$$\text{Total Distance} = \Delta X_1 + \Delta X_2 + \Delta X_3 = \boxed{1757.5\,m}$$

(Bonus) A ninja hound may run a total distance of 51.749 kilometers from where it was summoned by Kakashi. Let us suppose Kakashi gives it instructions to run along a spiral with equations $x = t\cos(t)$, $y = t\sin(t)$ (these are implicitly in kilometers and minutes). How long does Kakashi's hound run before it must return to the dog world from whence it came? Assume the motion starts at $t = 0$. You may use technology to perform the needed integration.

$$\vec{r}(t) = \langle t\cos t,\ t\sin t \rangle$$

$$\vec{v}(t) = \langle \cos t - t\sin t,\ \sin t + t\cos t \rangle$$

$$v = \sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2}$$

$$s(t) = \int_0^t \sqrt{(\cos\tau - \tau\sin\tau)^2 + (\sin\tau + \tau\cos\tau)^2}\, d\tau$$

$$= \int_0^t \sqrt{\underline{\cos^2\tau} - \cancel{2\tau\cos\tau\sin\tau} + \underline{\tau^2\sin^2\tau} + \underline{\sin^2\tau} + \cancel{2\tau\sin\tau\cos\tau} + \underline{\tau^2\cos^2\tau}}\, d\tau$$

$$= \int_0^t \sqrt{1 + \tau^2}\, d\tau$$

$\tau = \sinh\phi$

$d\tau = \cosh\phi\, d\phi$

$1 + \tau^2 = 1 + \sinh^2\phi = \cosh^2\phi$

$t = \sinh\phi_1$

$$= \int_0^{\phi_1} \cosh^2\phi\, d\phi$$

$$= \frac{1}{2}\int_0^{\phi_1} (1 + \cosh(2\phi))\, d\phi$$

$$= \frac{1}{2}\left(\phi_1 + \frac{1}{2}\sinh(2\phi_1)\right)$$

$$= \frac{1}{2}\left(\sinh^{-1}(t) + t\sqrt{1+t^2}\right)$$

$\phi_1 = \sinh^{-1}(t)$

$\sinh(2\phi_1) = 2\sinh\phi_1\cosh\phi_1$

$\cosh\phi_1 = \sqrt{1+t^2}$

$$51.749 = \frac{1}{2}\left(\sinh^{-1}(t) + t\sqrt{1+t^2}\right)$$

$$51.749 = \int_0^t \sqrt{1+\tau^2}\, d\tau \Rightarrow \boxed{t \cong 10.00\text{s}}$$