Your solutions should be neat, correct and complete. Same instructions as Mission 1 apply here.

Recommended Homework from Textbook (Serway):
Chapter 4 #'s 9, 11, 13, 19, 23, 25, 26, 50 & Chapter 5 #'s 5, 7, 11, 12

Recommended Homework from Recommended Textbook (Young & Freedman, 9th ed):
Chapter 3 (two dimensional motion, projectiles, circular motion, relative motion)
#'s 1, 4, 13, 15, 17, 19, 27, 29, 33, 37, 39, 43, 45, 49, 50, 51, 53, 55, 57, 59, 61, 63, 67, 71
Chapter 4 (Newton's Laws)
#'s 1, 5, 7, 9, 17, 21, 23, 27, 29, 31, 33, 37, 39, 43, 49, 51, 53

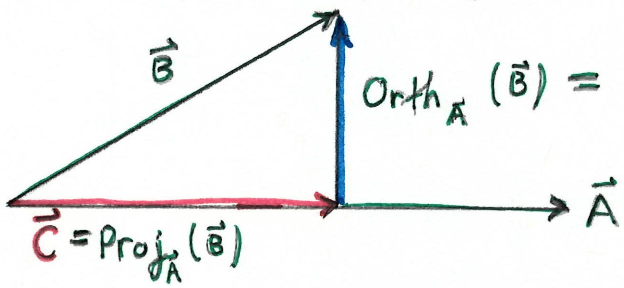
Suggested Reading the following resources may be helpful:

(a.) Lectures 5, 6, 7 as posted on the course website,

(b.) Chapters 2, 4, 5, 6 of the required text.

Problem 13: (2pts) Suppose $\vec{A} = \langle 1,2,2\rangle$ and $\vec{B} = \langle -2,0,7\rangle$. Find $\vec{C}, \vec{D}$ such that $\vec{B} = \vec{C} + \vec{D}$ where $\vec{C} \cdot \vec{D} = 0$ and $\vec{C}$ is colinear to $\vec{A}$.

You can solve this algebraically, but geometry is easier



$\mathrm{Orth}_{\vec{A}}(\vec{B}) = \vec{B} - \mathrm{Proj}_{\vec{A}}(\vec{B}) = \vec{D}$

$\hat{A} = \frac{1}{3}\langle 1,2,2\rangle$

$$\vec{C} = \mathrm{Proj}_{\vec{A}}(\vec{B}) = (\vec{B}\cdot\hat{A})\hat{A} = \frac{1}{9}(\langle -2,0,7\rangle\cdot\langle 1,2,2\rangle)\langle 1,2,2\rangle$$
$$= \frac{1}{9}(-2+0+14)\langle 1,2,2\rangle$$
$$= \boxed{\langle 4/3, 8/3, 8/3\rangle = \vec{C}}$$

Then, $\vec{D} = \mathrm{Orth}_{\vec{A}}(\vec{B}) = \langle -2,0,7\rangle - \langle 4/3, 8/3, 8/3\rangle = \boxed{\left\langle \frac{-10}{3}, \frac{-8}{3}, \frac{13}{3}\right\rangle = \vec{D}}$

We can check, $\vec{C} = \underbrace{\langle 4/3, 8/3, 8/3\rangle = \frac{4}{3}\langle 1,2,2\rangle}_{\text{colinear with } \langle 1,2,2\rangle = \vec{A}}$

and

$$\vec{C}\cdot\vec{D} = \frac{4}{9}\langle 1,2,2\rangle\cdot\langle -10,-8,13\rangle = \frac{4}{9}(-10-16+26) = 0$$

and as requested $\vec{C} + \vec{D} = \langle -2, 0, 7\rangle$.

Problem 14: (2pts) Suppose $\vec{c}$ is a constant vector. Further, suppose the initial position of a particle is $\vec{r}_o$ and the initial velocity is $\vec{v}_o$ at time $t = 0$. Given that the acceleration $\vec{a} = t\vec{c}$, find the velocity and position as a function of time t in terms of the given vectors.

$$\vec{a} = t\vec{c} = \frac{d\vec{V}}{dt} \Rightarrow \int_0^t \frac{d\vec{V}}{d\tau}\,d\tau = \int_0^t \tau\,\vec{c}\,d\tau$$

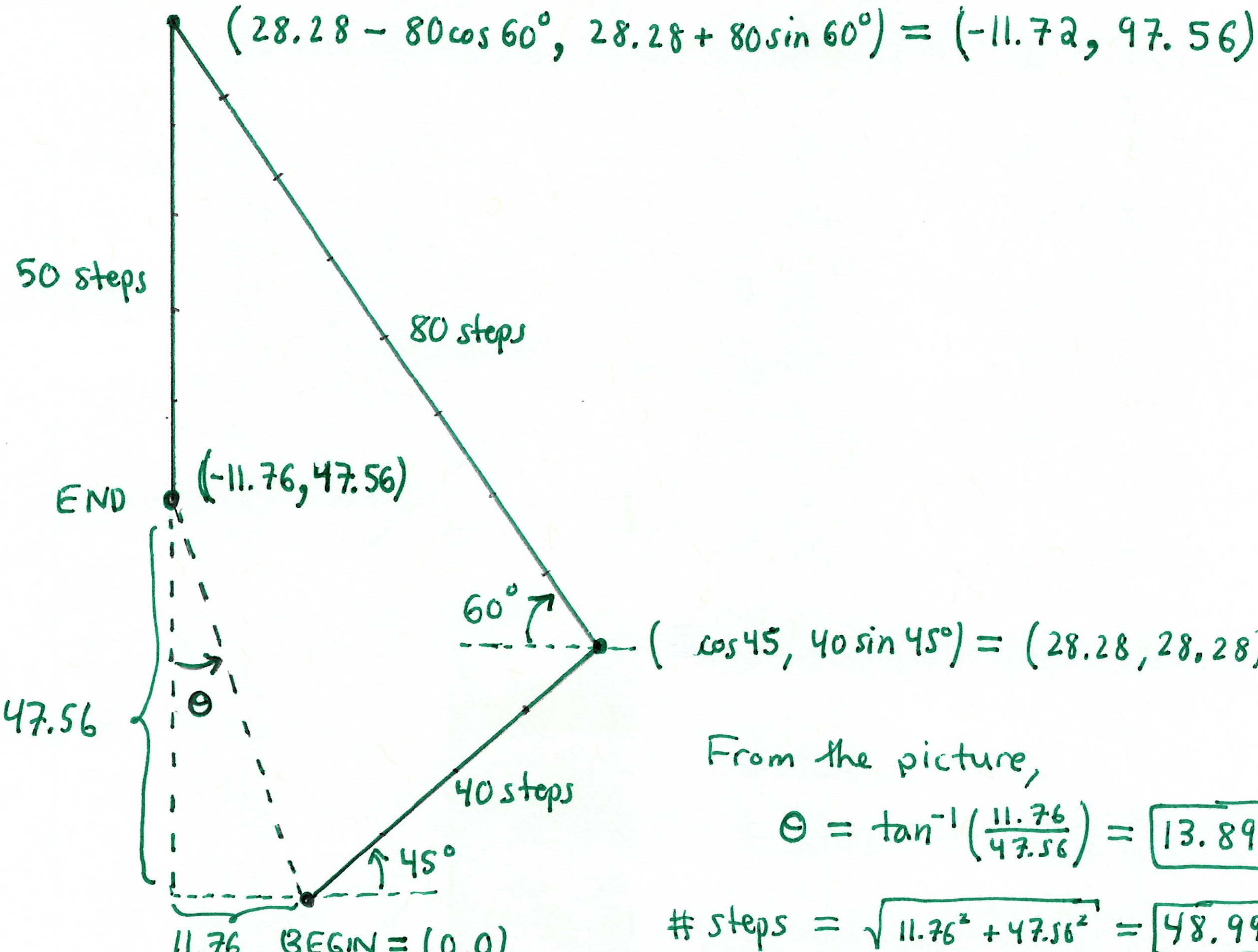
$$\vec{V}(t) - \vec{V}(0) = \left(\int_0^t \tau\,d\tau\right)\vec{c} = \frac{1}{2}t^2\vec{c}$$

$$\therefore \boxed{\vec{V}(t) = \vec{V}_o + \frac{1}{2}t^2\vec{c}}$$

$$\int_0^t \frac{d\vec{r}}{d\tau}\,d\tau = \int_0^t \left(\vec{V}_o + \frac{1}{2}\tau^2\vec{c}\right)d\tau \Rightarrow \vec{r}(t) - \vec{r}(0) = \vec{V}_o t + \frac{1}{6}t^3\vec{c}$$

$$\therefore \boxed{\vec{r}(t) = \vec{r}_o + \vec{V}_o t + \frac{1}{6}t^3\vec{c}}$$

Problem 15: (2pts) A ninja wanders through a dense cloud of hidden mist. He takes 40 steps northeast, then 80 steps 60^o north of west, then 50 steps due south. Assuming he is facing due south at the end, tell him by what angle he should rotate Counter-Clock-Wise(CCW) before walking straight to return to his initial starting point. Also, how many steps should will he need to return to the starting point? (answers of the form, he's a ninja so he can just jump, glide, etc... whatever, will be amusing, but will not earn points)



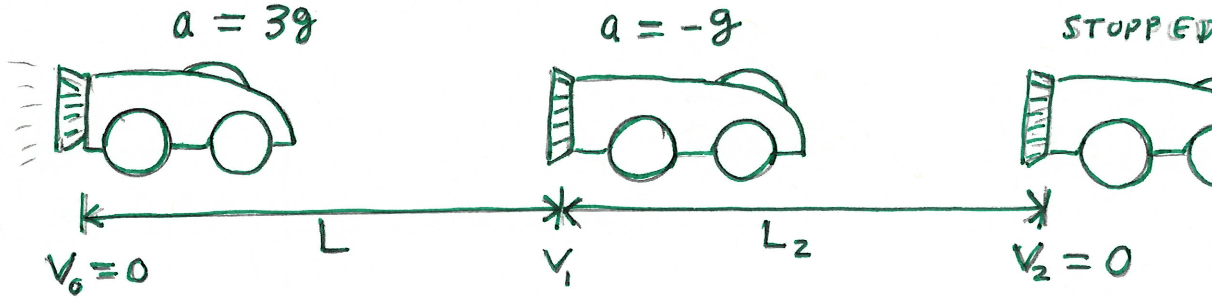
From the picture,

$$\theta = \tan^{-1}\left(\frac{11.76}{47.56}\right) = \boxed{13.89^\circ}$$

$$\#\text{ steps} = \sqrt{11.76^2 + 47.56^2} = \boxed{48.99}$$

(so, apparently 49 steps)

Problem 16: (2pts) A rocket car accelerates at $a = 3g$ over a distance of L. Then the car applies brakes which give $a = -g$ until the car comes to rest. Find the total distance the car travels. Your answer should be in terms of the given distance L.

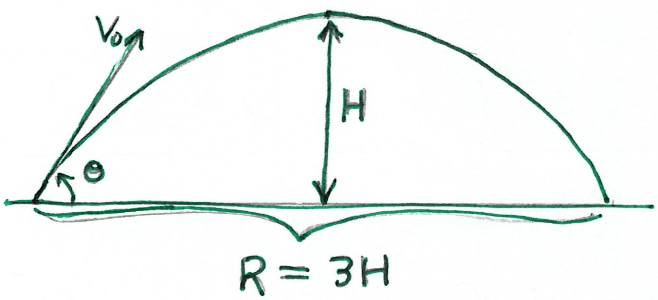


$$V_1^2 = V_0^2 + 2(3g)L \Rightarrow \underline{V_1^2 = 6gL}.$$

$$V_2^2 = V_1^2 + 2(-g)L_2 \Rightarrow 0 = V_1^2 - 2gL_2 \Rightarrow L_2 = \frac{V_1^2}{2g} = \frac{6gL}{2g}$$

Thus $L_2 = 3L$ and we find total distance $= L + L_2 = \boxed{4L}$

Problem 17: (2pts) A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. At what angle of inclination was the projectile fired. Assume a level landscape and ignore air friction.



We derived

$$H = \frac{V_0^2 \sin^2\theta}{2g} \quad \text{(max height)}$$

$$R = \frac{V_0^2 \sin(2\theta)}{g} = \frac{2V_0^2 \sin\theta\cos\theta}{g}$$

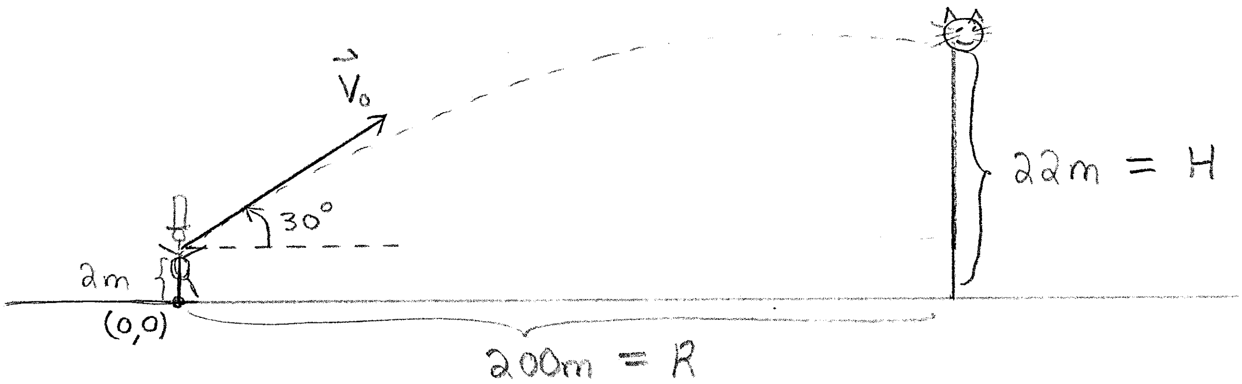
Thus, $R = 3H \Rightarrow \dfrac{2V_0^2 \sin\theta\cos\theta}{g} = \dfrac{3V_0^2 \sin^2\theta}{2g}$

$$\Rightarrow \frac{\sin\theta\cos\theta}{\sin\theta\sin\theta} = \frac{3}{4}$$

$$\Rightarrow \tan\theta = \frac{4}{3}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right) = \boxed{53.13^\circ}$$

Problem 18: (2pts) Imagine you shoot an arrow at a speed of v_o at an angle of inclination of $\theta = 30^o$. If the arrow leaves the bow at a height of $2.0\,m$ above the ground and you are trying to shoot a cat (it's evil, in case you're worried) in a tree $200\,m$ away in a branch $22\,m$ above the ground. Find the speed v_o needed in order to shoot the cat.



$$x = (V_o \cos\theta)t \qquad \theta = 30^o$$

$$y = y_o + (V_o \sin\theta)t - \frac{1}{2}gt^2 \qquad y_o = 2m$$

For the time t when the cat problem is solved we have

$x = (V_o\cos\theta)t = R$ and $\underline{y = y_o + (V_o\sin\theta)t - \frac{1}{2}gt^2 = H}$ *

Thus, $t = \dfrac{R}{V_o\cos\theta}$ which substituted into * yields,

$$y_o + (V_o\sin\theta)\left(\frac{R}{V_o\cos\theta}\right) - \frac{1}{2}g\left(\frac{R}{V_o\cos\theta}\right)^2 = H$$

$$y_o + R\tan\theta - \frac{gR^2}{2\cos^2\theta}\frac{1}{V_o^2} = H$$

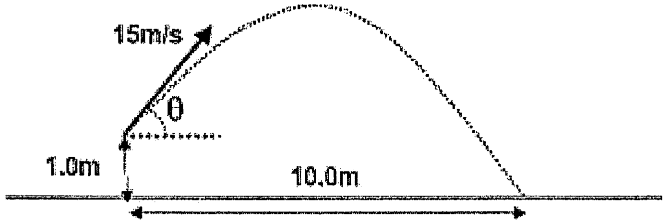
Solve for V_o,

$$y_o - H + R\tan\theta = \frac{gR^2}{2\cos^2\theta}\frac{1}{V_o^2}$$

$$V_o = \frac{R}{\cos\theta}\sqrt{\frac{g}{2(y_o - H + R\tan\theta)}}$$

$$\boxed{V_o = 52.32\ m/s}$$

Problem 19: (2pts) Ron Swanson mistakedly orders a *sausage* sandwhich at a vegan run donut shop. After taking a bite he recoils in horror and throws the faux-meat entree at 15 m/s as shown below. At what θ did Ron Swanson throw the pathetic veggy sandwhich?



$R = 10.0\,m$

$y_0 = 1.0\,m$

$$X = (V_0 \cos\theta)t$$

$$y = y_0 + (V_0 \sin\theta)t - \frac{1}{2}gt^2$$

Fake meat hits ground at time t when $X = R$ and $y = 0$

$$R = (V_0 \cos\theta)t \quad \Rightarrow \quad t = \frac{R}{V_0 \cos\theta}$$

$$0 = y_0 + (V_0 \sin\theta)t - \frac{1}{2}gt^2$$

$$0 = y_0 + (V_0 \sin\theta)\left(\frac{R}{V_0 \cos\theta}\right) - \frac{1}{2}g\left(\frac{R}{V_0 \cos\theta}\right)^2$$

Notice, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sec\theta = \dfrac{1}{\cos\theta}$ thus,

$$y_0 + R\tan\theta - \frac{gR^2}{2V_0^2}\sec^2\theta = 0$$

$$\sec^2\theta - \frac{2V_0^2 R}{gR^2}\tan\theta - \frac{2V_0^2 y_0}{gR^2} = 0$$

$$\tan^2\theta - \frac{2V_0^2}{gR}\tan\theta + 1 - \frac{2V_0^2 y_0}{gR^2} = 0 \quad *$$

Let $\lambda = \tan\theta$ and note $\dfrac{2V_0^2}{gR} \cong 4.5918$ and $1 - \dfrac{2V_0^2 y_0}{gR^2} = 0.54082$ thus $*$ gives quadratic eq$^{\underline{n}}$

$\lambda^2 - 4.5918\lambda + 0.54082 = 0$ which has solutions of $\lambda_1 = 4.4708$ and $\lambda_2 = 0.12097$. Thus $\theta = \tan^{-1}(\lambda)$,

$\boxed{\theta_1 = 77.39^\circ}$ or $\boxed{\theta_2 = 6.898^\circ}$

Problem 20: (2pts) Consider coordinate systems (x_1, x_2, x_3) and (y_1, y_2, y_3) and (z_1, z_2, z_3). Suppose a given particle has the following trajectories as measured by the x, y and z observers respective:

$$(x_1, x_2, x_3) = (1 - t,\ 2 + 2t,\ 3 - 7t + t^2)$$
$$(y_1, y_2, y_3) = (3t,\ 4,\ 8t + (t-1)^2)$$
$$(z_1, z_2, z_3) = (4 - t,\ 4t,\ t^3)$$

(a.) Calculate velocities with respect to the given coordinate systems: $\vec{v}_X, \vec{v}_Y, \vec{v}_Z$.

(b.) Calculate accelerations with respect to the given coordinate systems: $\vec{a}_X, \vec{a}_Y, \vec{a}_Z$.

(c.) Suppose that (x_1, x_2, x_3) is an inertial coordinate system. Which of the other coordinate systems *could* be inertial as well ?

(a.) $\vec{V}_X = \frac{d}{dt}\langle 1-t,\ 2+2t,\ 3-7t+t^2\rangle = \langle -1,\ 2,\ -7+2t\rangle$

$\vec{V}_Y = \frac{d}{dt}\langle 3t,\ 4,\ 8t+(t-1)^2\rangle = \langle 3,\ 0,\ 8+2(t-1)\rangle$

$\vec{V}_Z = \frac{d}{dt}\langle 4-t,\ 4t,\ t^3\rangle = \langle -1,\ 4,\ 3t^2\rangle$

(b.) $\vec{a}_X = \dfrac{d\vec{V}_X}{dt} = \langle 0, 0, 2\rangle$

$\vec{a}_Y = \dfrac{d\vec{V}_Y}{dt} = \langle 0, 0, 2\rangle$

$\vec{a}_Z = \dfrac{d\vec{V}_Z}{dt} = \langle 0, 0, 6t\rangle$

(c.) since $\vec{a}_X \neq \vec{a}_Z$ these are not inertially related. However $\vec{a}_X = \vec{a}_Y$ so it is possible X and Y are inertially related, hence Y could be an inertial coord. system.

Remark: here I truncated the meaning of inertial coord. systems to not include rotated coordinates. The complete discussion requires rotation matrices...

Problem 21: (2pts) Suppose two observers (x_1, x_2) and (y_1, y_2) are related at time t according to

$$(y_1, y_2) = (t + \cos t + x_1, t + \sin t + x_2)$$

(a.) Show that the observes are not inertially related.

(b.) Suppose $(x_1, x_2) = (1 - t, 2 - t)$. Find the trajectory in (y_1, y_2) and describe the geometry of the trajectory. Is this object in *uniform rectilinear motion* ? Why is this a tricky question ?

(c.) Suppose (x_1, x_2) is an inertial coordinate system in which the net-force on a particle with mass m is measured to be zero. Find the acceleration of that particle in the non-inertial frame (y_1, y_2).

(a.) $(y_1, y_2) = (x_1, x_2) + t\langle 1,1\rangle + \underbrace{\langle \cos t, \sin t\rangle}_{\text{not allowed.}}$

We needed $\bar{Y} = \bar{X} + \vec{r}_o + t\vec{V}_o$

and the term $\langle \cos t, \sin t\rangle$ does not coincide.

Also, can calculate by differentiating,

$$\vec{V}_{\bar{Y}} = \vec{V}_{\bar{X}} + \langle 1,1\rangle + \langle -\sin t, \cos t\rangle$$

$$\vec{a}_{\bar{Y}} = \vec{a}_{\bar{X}} + \langle -\cos t, -\sin t\rangle$$

thus $\vec{a}_{\bar{Y}} \neq \vec{a}_{\bar{X}}$.

(b.) If $(x_1, x_2) = (1-t, 2-t)$ then

$$(y_1, y_2) = (t + \cos t + 1 - t, t + \sin t + 2 - t)$$

$$= \underbrace{(1 + \cos t, 2 + \sin t)}$$

this parametrizes a circle of radius 1, centered at (1,2) in the CCW direction in the $\bar{Y}$-coord.

- The object is in uniform rectilinear motion with respect to the $\bar{X}$-coord. system, so YES.
- it's tricky because $\bar{Y}$-coord. system is an <u>accelerated</u> frame of reference, the circular motion in $\bar{Y}$ is a "frame-effect".

PROBLEM 21 continued

(c.) If $\vec{a}_X = 0$ then by the calculations in (a.) we note

$$\vec{a}_Y = \vec{a}_X + \langle -\cos t, -\sin t \rangle$$

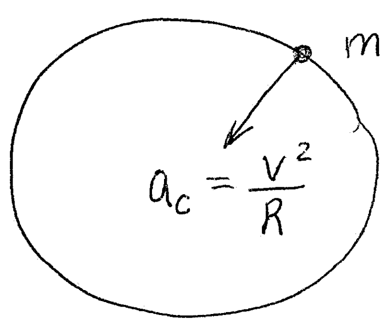
$$\therefore \boxed{\vec{a}_Y = \langle -\cos t, -\sin t \rangle}$$

Remark: if you can imagine using Y-coord. with the false assumption Y are inertial coord. then apply Newton's 2nd Law to find,

$$m\vec{a}_Y = \langle -m\cos t, -m\sin t \rangle = \vec{F}_{net}$$

of course, there isn't actually a force on m, this is just a fictional force which has appeared because Y is rotating

Similar, but more common case

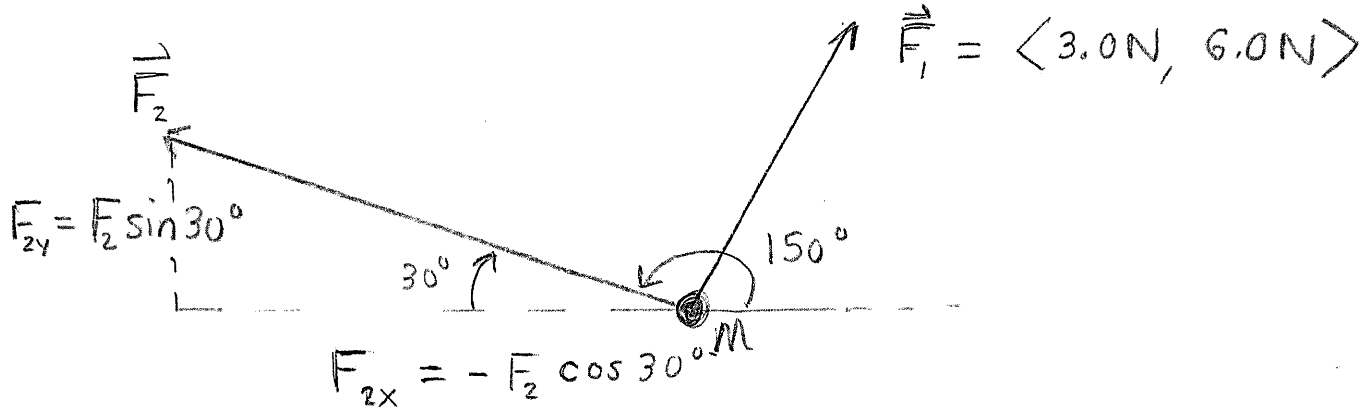


- m going in circle in inertial coordinates
- m fixed in place in comoving coordinates must have $ma_c = mv^2/R$

Problem 22: (2pts) Let $\hat{u}$ be a unit-vector directed 30^o north of west. Suppose forces

$$\vec{F}_1 = (3.0\,N)\hat{\mathbf{x}} + (6.0\,N)\hat{\mathbf{y}} \qquad \& \qquad \vec{F}_2 = (20\,N)\hat{u}$$

act on a body with mass $M = 10\,kg$. Find the magnitude and direction of the resulting acceleration of the mass.



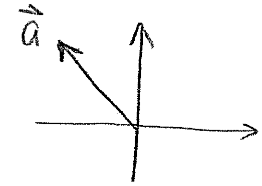
$$\vec{F}_2 = \langle -F_2 \cos 30^\circ, F_2 \sin 30^\circ \rangle = \langle -17.32\,N, 10.0\,N \rangle$$

Or, notice $\hat{u} = \langle \cos 150^\circ, \sin 150^\circ \rangle$ and thus $\vec{F}_2 = 20N \langle -0.866, 0.5 \rangle$ which gives

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$
$$= \langle 3.0\,N, 6.0\,N \rangle + \langle -17.32\,N, 10.0\,N \rangle$$
$$= \langle -14.32\,N, 16.0\,N \rangle$$

Then Newton's 2^{nd} Law gives $\vec{F}_{net} = M\vec{a}$ where $M = 10\,kg$ thus $\vec{a} = \frac{1}{10\,kg} \langle -14.32N, 16.0N \rangle$

$$\therefore \quad \vec{a} = \langle -1.432\,m/s^2, 1.6\,m/s^2 \rangle$$



We calculate $a = \sqrt{\vec{a} \cdot \vec{a}} = \boxed{2.147\,m/s^2}$

and $\Theta = 180^\circ - \tan^{-1}\left(\frac{1.6}{1.432}\right) = \boxed{131.8^\circ}$ direction of $\vec{a}$

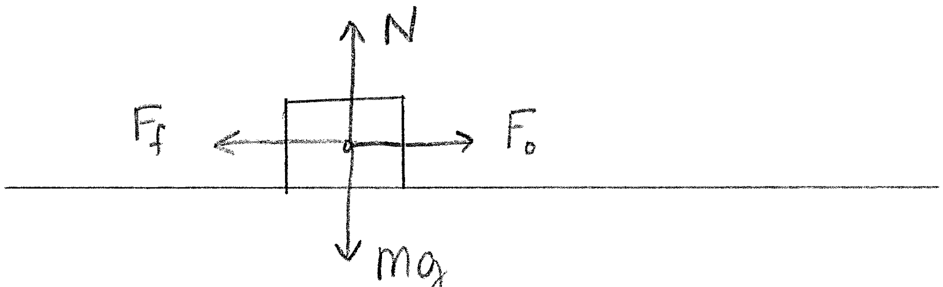
(you could also have given $\hat{a} = \langle -0.6665, 0.7455 \rangle$)

Problem 23: (2pts) Consider a box with mass $m = 20\,kg$ which rests on a floor with coefficient of static friction $\mu_s = 0.7$ and coefficient of kinetic friction $\mu_k = 0.5$.

(a.) If F_o is applied horizontally to the box, then what is the maximum force which can be applied without the box moving ?

(b.) Supposing the box is given a nudge to start the box sliding, then what is the acceleration of the box if it continues to be pushed with force F_o horizontally after it begins moving ?

(a.)



$$ma_y = 0 = N - mg \Rightarrow \underline{N = mg}.$$

$$ma_x = F_o - F_f = 0 \Rightarrow F_o = F_f \leq \mu_s N$$

Consequently, $F_o = \mu_s mg$ is max-force to keep box motionless.

$$F_{o,max} = (0.7)(20kg)(9.8\tfrac{m}{s^2}) = \boxed{137.2N}$$

(b.) $ma_y = 0 = N - mg$ (box on plane)

$ma_x = F_o - F_f$ (kinetic friction, $F_f = \mu_k N$)

$$ma_x = \mu_s mg - \mu_k mg$$

$$a_x = (\mu_s - \mu_k)g = \boxed{1.96\ m/s^2}$$

<u>Remark:</u> we can answer (b.) without knowing the actual value of m.

<u>Alternative Sol^n:</u> $F_{f,kinetic} = (0.5)(20kg)(9.8\frac{m}{s^2}) = 98N$

$F_{net} = 137.2N - 98N = 39.2N$

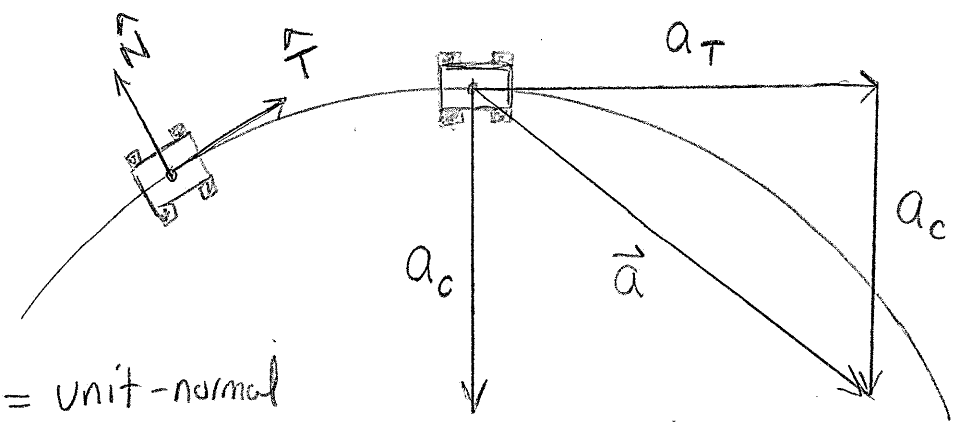
$\rightarrow a_x = \dfrac{39.2N}{20kg} = 1.96\ m/s^2$

Problem 24: (2pts) A car travels on a flat circular track of radius $R = 50\,m$ with wheels that have a coefficient of friction of $\mu = 0.9$.

(a.) What is the maximum speed v possible for the car to stay on the track ?

(b.) At a particular time, the car is increasing its speed according to $dv/dt = 2m/s^2$. What is the maximum speed v the car have under such an acceleration if it is to stay on the track ?

Set-up



$\hat{N}$ = unit-normal

$\hat{T}$ = unit-tangent

$$\vec{a} = -a_c \hat{N} + a_T \hat{T} \quad \text{where} \quad \underbrace{a_c = \frac{V^2}{R}}_{\text{centripetal acceleration}} \quad \text{and} \quad \underbrace{a_T = \frac{dV}{dt}}_{\text{tangential acceleration}}$$

$$a = \sqrt{a_c^2 + a_T^2}$$

(a.) flat track, car on plane of motion $\Rightarrow$ $\underline{N = mg}$.

Thus $F_f = \mu N = 0.9\,mg$.

$$m\vec{a} = \vec{F}_f + \underbrace{\vec{N} + \vec{F}_{gravity}}_{\text{vertical forces cancel out}} = \vec{F}_f$$

max friction force available

$$\Rightarrow \quad ma = F_f = 0.9\,mg$$

$a_T = 0$

$$\Rightarrow \quad a = 0.9\,g = \sqrt{a_c^2 + a_T^2} = a_c$$

Clearly $a_T = 0$ for max speed, hence solve

$$0.9g = a_c = \frac{V^2}{R} \Leftrightarrow V_{max} = \sqrt{0.9Rg} \cong \boxed{21\ m/s}$$

PROBLEM 24 continued

(b.) If $\frac{dV}{dt} = 2\,\frac{m}{s^2} = a_T$ and $a_c = \frac{V^2}{R}$

where $R = 50m$ and we use max-friction force $F_f = \mu m g = 0.9mg$ then once more we have

$$a = 0.9g = \sqrt{a_c^2 + a_T^2}$$

$$a_c^2 = (0.9g)^2 - a_T^2 \cong 73.79\,\frac{m^2}{s^4}$$

$$a_c = \sqrt{73.79\,\frac{m^2}{s^4}} = 8.590\,\frac{m}{s^2}$$

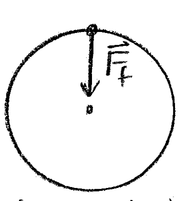
$$a_c = \frac{V^2}{R}$$

$$V = \sqrt{R a_c} = \sqrt{(50m)(8.590\,m/s^2)}$$

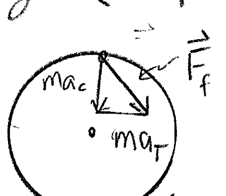
$$\boxed{V = 20.72\,m/s}$$

Remark: for (a.) notice $a_c = \frac{V_{max}^2}{R} = \frac{(21\,m/s)^2}{50m} \cong 8.82\,\frac{m}{s^2}$

now some of the $\vec{a}$ points tangentially so a_c reduces slightly (F_f maintaining motion same for (a.) & (b.)) magnitude, but to be clear the direction differs.



(part (a.))



(part (b.))