Your solutions should be neat, correct and complete. Same instructions as Mission 1 apply here.

Recommended Homework from Textbook (Serway):
Chapter 5 #'s 17, 19, 23, 29, 45, 49 & Chapter 6 #'s 15, 29, 31

Recommended Homework from Recommended Textbook (Young & Freedman, 9th ed):
Chapter 5 (application of Newton's Laws) #'s 1, 3, 5, 7, 9, 13, 15, 17, 19, 25, 27, 31, 33, 35, 37, 39, 41, 51, 53, 55, 63, 65, 67, 73, 75, 77, 79, 87, 91, 95, 97, 101, 110

Suggested Reading the following resources may be helpful:

(a.) Lectures 8, 9, 10, 11, 13 as posted on the course website,

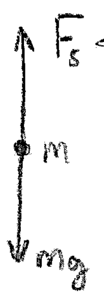
(b.) Chapters 5 and 6 of the required text.

Problem 25: (2pts) A 55 kg physics student stands on a bathroom scale in an elevator. As the elevator starts moving, the scale reads 45 kg. For the questions below take up to be the positive direction.

(a.) Find the acceleration of the elevator.

(b.) What is the acceleration if the scale reads 67 kg ?

(c.) If the scale reads zero, what is the acceleration of the elevator ? Should the student worry in this case ?



$F_s \leftarrow$ matches weight read by scale ; $F_s = (45\,kg)(9.8\,\frac{m}{s^2}) = \underline{441\,N}$

$ma = F_s - mg$ where $m = 55\,kg$

$a = \dfrac{441N - mg}{m} = \dfrac{441N}{55kg} - 9.8\,\frac{m}{s^2} = -1.78\,\frac{m}{s^2}$

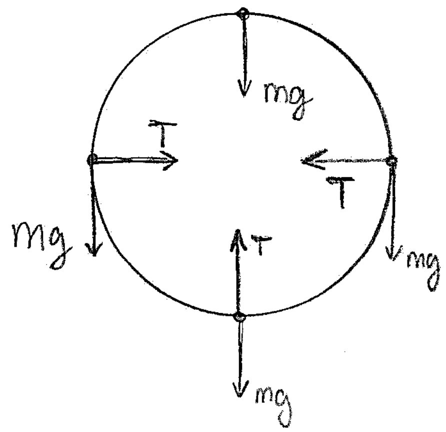
(a.) the acceleration $\boxed{a = -1.78\,m/s^2}$

(b.) $a = \dfrac{(67kg)(9.8\,m/s^2)}{55\,kg} - 9.8\,m/s^2 = \boxed{2.138\,m/s^2}$

(c.) $ma = F_s - mg$ and $F_s = 0 \Rightarrow \boxed{a = -g = -9.8\,m/s^2}$

YES, THE STUDENT IS IN FREE FALL, WORRY IS WARRANTED.

Problem 26: (1pts) A cord is tied to a pail of water, and the pail is swung in a vertical circle of radius $1.200\,m$. What must be the minimum speed of the pail at the highest point of the circle if no water is to spill from it?



$$m\vec{a} = \vec{F}_{gravity} + \vec{T}$$

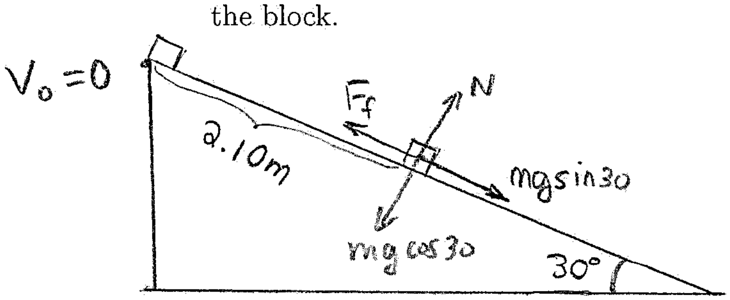
at the highest point, the minimum speed is given when $T = 0$ and $\vec{a}$ is center-seeking ($a_T = 0$).

$$ma = m\left(\frac{v^2}{R}\right) = mg$$

$$v = \sqrt{gR} = \sqrt{(9.8\,\tfrac{m}{s^2})(1.2\,m)}$$

$$\boxed{v = 3.429\ m/s}$$

Problem 27: (1pts) A $2.00\,kg$ block starts from rest at the top of a 30^o incline and slides a distance of $2.10\,m$ down the incline in $1.70\,s$. Find the magnitude of the frictional force acting on the block.



$$ma = mg\sin 30 - F_f$$

$$V_{avg} = \frac{V_o + V_f}{2} = \frac{2.10\,m}{1.70\,s}$$

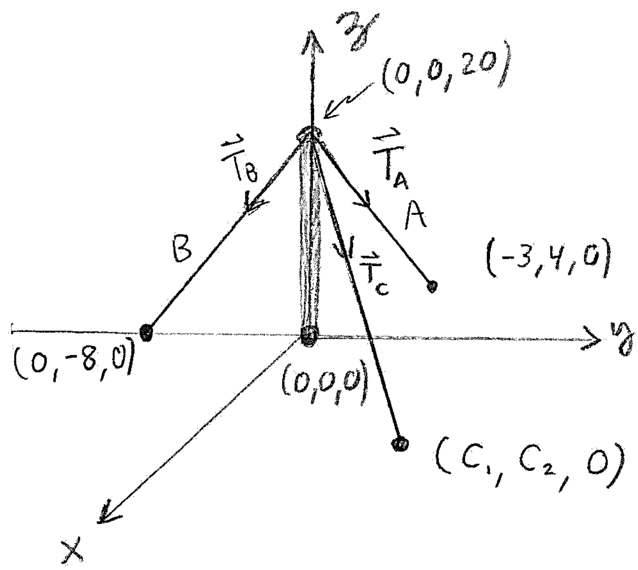
$$V_f = 2\left(\frac{2.10\,m}{1.70\,s}\right) = 2.471\ m/s$$

However, we also know $V_f^2 = V_o^2 + 2a(2.10\,m) = (2.471\ m/s)^2$

Thus, $a = \dfrac{(2.471\ m/s^2)^2}{2(2.10\,m)} = 1.453\ m/s^2$. Solve Newton's 2nd Law for $F_f = m(g\sin 30° - a) = \boxed{6.893\ N}$

vertical

Problem 28: (2pts) A tent pole is fastened with three cables. The pole is $20\,m$ tall and let us place the base of the pole at the origin $(0,0,0)$ in meters. Further, suppose cable A is secured to the point $(-3,4,0)m$ and cable B is secured to the point $(0,-8,0)$. Both cable A and B have tension $1000\,N$. If cable C is also to be anchored to the xy-plane at some point $(c_1,c_2,0)$ and have a tension of $1000\,N$ then where must we anchor the third cable in order to keep the pole from falling over ? Also, what is the normal force of the ground on the pole if the pole has a mass of $40\,kg$?



$$T_A = T_B = T_C = 1000\,N$$

$$\vec{A} = (-3,4,0)-(0,0,20) = \langle -3, 4, -20\rangle$$
$$\vec{B} = (0,-8,0)-(0,0,20) = \langle 0,-8,-20\rangle$$
$$\vec{C} = (c_1,c_2,0)-(0,0,20) = \langle c_1, c_2, -20\rangle$$

these vectors point in directions of the tensions of cables A, B, C respective

$$\vec{T}_A = T_A\hat{A} = \frac{T_A}{20.61}\langle -3,4,-20\rangle$$
$$\vec{T}_B = T_B\hat{B} = \frac{T_B}{21.54}\langle 0,-8,-20\rangle$$
$$\vec{T}_C = T_C\hat{C} = \frac{T_C}{\sqrt{c_1^2+c_2^2+400}}\langle c_1, c_2, -20\rangle$$

equilibrium, pole not moving.

$$m\vec{a} = m\langle 0,0,-9.8\,m/s^2\rangle + \langle 0,0,f_N\rangle + \vec{T}_A + \vec{T}_B + \vec{T}_C = 0$$

We may focus on xy - components to solve for c_1 & c_2,

$$x|\quad \frac{T_A}{20.61}(-3) + \frac{T_C\, c_1}{\sqrt{c_1^2+c_2^2+400}} = 0 \Rightarrow \frac{c_1}{\sqrt{c_1^2+c_2^2+400}} = \frac{3}{20.61}$$

$$y|\quad \frac{T_A}{20.61}(4) + \frac{T_B}{21.54}(-8) + \frac{T_C\cdot c_2}{\sqrt{c_1^2+c_2^2+400}} = 0 \Rightarrow \frac{c_2}{\sqrt{c_1^2+c_2^2+400}} = 0.1773$$

We find $c_1 = 0.1456\sqrt{c_1^2+c_2^2+400}$ & $c_2 = 0.1773\sqrt{c_1^2+c_2^2+400}$

Thus $\sqrt{c_1^2+c_2^2+400} = \dfrac{c_1}{0.1456} = \dfrac{c_2}{0.1773} \Rightarrow \underline{c_2 \cong 1.2177\,c_1}$

PROBLEM 28 continued

$$C_1 = 0.1456\sqrt{C_1^2 + C_2^2 + 400} \quad \& \quad C_2 = 1.2177\,C_1$$

$$C_1^2 = (0.1456)^2\left(C_1^2 + (1.2177)^2 C_1^2 + 400\right)$$

$$C_1^2\left(1 - (0.1456)^2 - (0.1456)^2(1.2177)^2\right) = (0.1456)^2(400)$$

$$C_1^2 = \frac{8.4797}{0.94737} \cong 8.951$$

$$\Rightarrow C_1 = 2.992 \quad \& \quad C_2 = 3.643$$

We should anchor the 3rd cable at $\boxed{(2.992,\ 3.643,\ 0)\,m}$

Then $\sqrt{C_1^2 + C_2^2 + 400} \cong 20.54$ so returning to z-component of Newton's 2nd Law,

$$0 = -mg + f_N - \frac{20\,T_A}{20.61} - \frac{20\,T_B}{21.54} - \frac{20\,T_C}{20.54}$$

$T_A = T_B = T_C$ ALL 1000N

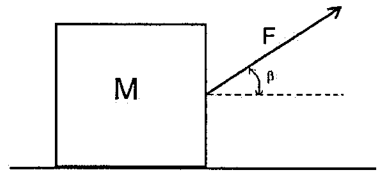
$$f_N = mg + \left(\frac{20}{20.61} + \frac{20}{21.54} + \frac{20}{20.54}\right)(1000N)$$

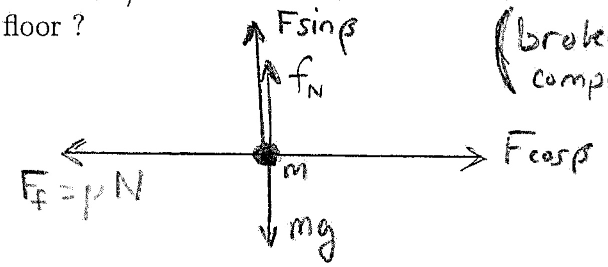
$$f_N = (40kg)(9.8\tfrac{m}{s^2}) + (2.873)(1000N)$$

$$\boxed{f_N = 3265\,N}$$

f_N = normal force

Problem 29: (1pts) Suppose a mass $M = 20\,kg$ is pulled by a force $F = 100\ N$ at angle $\beta = 50^o$ as pictured. If the box accelerates at $a = 2.0 m/s^2$ then what is the **coefficient of kinetic friction** between the box and the floor ?





(broke F into components)

$$\parallel: \quad ma = F\cos\beta - \mu f_N$$

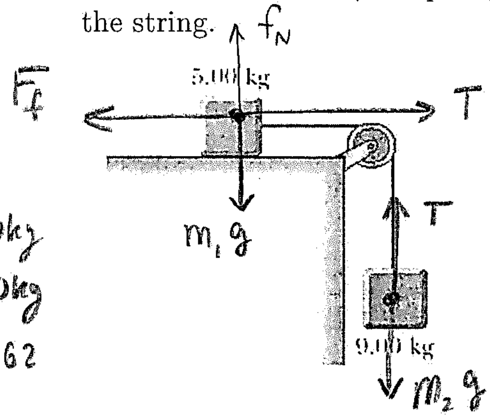
$$\perp: \quad 0 = F\sin\beta + f_N - mg \quad \Rightarrow \quad f_N = mg - F\sin\beta = 119.4\ \frac{kg\,m}{s^2}$$

$$\mu f_N = F\cos\beta - ma$$

$$\mu = \frac{F\cos\beta - ma}{f_N} = \frac{(100N)(\cos 50^\circ) - (20kg)(2.0\,m/s^2)}{119.4\ kg\,m/s^2}$$

$$\boxed{\mu \cong 0.2033}$$

Problem 30: (2pts) A 9.00 kg hanging weight is connected by a string over a pulley to a 5.00 kg block that is sliding on a flat table with coefficient of kinetic friction 0.162. The string is light and does not stretch; the pulley is light and turns without friction. Find the tension in the string.



$m_1 = 5.00kg$
$m_2 = 9.00kg$
$\mu = 0.162$

$$m_1 a = T - F_f \quad \& \quad f_N = m_1 g$$

$$m_1 a = T - \mu m_1 g \quad (1)$$

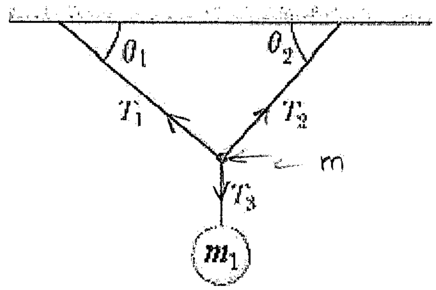
$$m_2 a = m_2 g - T \quad (2)$$

Solve both (1) and (2) for $a = \dfrac{T - \mu m_1 g}{m_1} = \dfrac{m_2 g - T}{m_2}$

$$\left(\frac{1}{m_1} + \frac{1}{m_2}\right) T = \mu g + g$$

$$T = \frac{(\mu + 1) g}{\frac{1}{m_1} + \frac{1}{m_2}} = \frac{m_1 m_2 (\mu + 1) g}{m_2 + m_1} = \boxed{36.60\ N}$$

Problem 31: (2pts) Suppose $m_1 = 3\,kg$ is suspended by ropes as pictured. Given $\theta_1 = 39^o$ and $\theta_2 = 51^o$, find the values of T_1, T_2 and T_3.



$T_{2x} = T_2 \cos\theta_2$

$T_{2y} = T_2 \sin\theta_2$

$T_{1x} = -T_1 \cos\theta_1$

$T_{1y} = T_1 \sin\theta_1$

$\boxed{T_3 = m_1 g = 29.4\,N}$

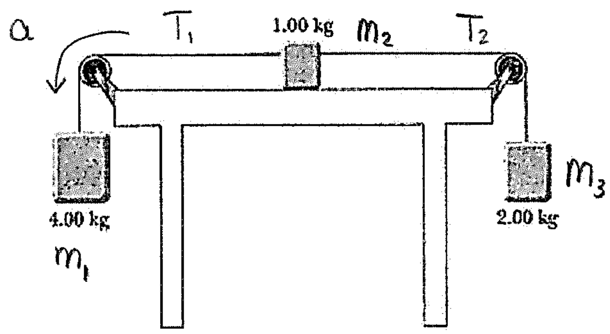
Equilibrium: $m\vec{a} = 0$

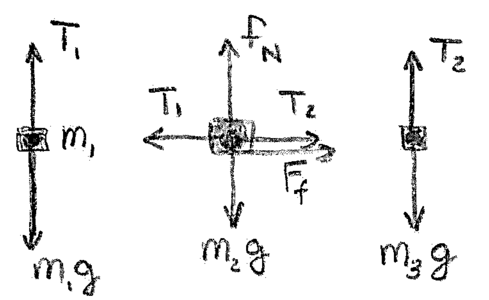
$x|\quad T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0 \quad \Rightarrow \quad T_2 = \frac{\cos\theta_1}{\cos\theta_2} T_1$

$y|\quad T_1 \sin\theta_1 + T_2 \sin\theta_2 - m_1 g = 0 \quad \Rightarrow \quad T_1 \sin\theta_1 + \left(\frac{\cos\theta_1}{\cos\theta_2}\right) T_1 \sin\theta_2 = m_1 g$

$$T_1 = \frac{29.4N}{\sin 39^\circ + \frac{\cos 39^\circ}{\cos 51^\circ}\sin 51^\circ} = \boxed{18.50\,N} \;\&\; \boxed{T_2 = 22.85\,N}$$

Problem 32: (2pts) Three blocks are connected on the table as shown below. The table is rough and has a coefficient of kinetic friction of 0.360. The objects have masses of 4.00 kg, 1.00 kg and 2.00 kg, as shown, and the pulleys are frictionless. Find the acceleration of the 1.00 kg block and the tensions of the left and right cords.





$f_N = m_2 g \rightarrow F_f = \mu m_2 g$

$$\left.\begin{aligned} m_1 a &= m_1 g - T_1 \\ m_2 a &= T_1 - T_2 - F_f \\ m_3 a &= T_2 - m_3 g \end{aligned}\right\} (m_1 + m_2 + m_3) a = m_1 g - F_f - m_3 g$$

$$a = \left(\frac{m_1 - \mu m_2 - m_3}{m_1 + m_2 + m_3}\right) g = \boxed{2.296\,m/s^2}$$

$$T_1 = m_1 g - m_1 a = m_1 (g - a) = (4.0kg)(7.504\,\tfrac{m}{s^2}) \cong \boxed{30.02\,N}$$

$$T_2 = m_3 a + m_3 g = m_3 (a + g) = (2.0kg)(12.096\,\tfrac{m}{s^2}) = \boxed{24.19\,N}$$

Problem 33: (3pts) Three blocks are in contact with one another on a frictionless, horizontal surface as shown. A horizontal force applied to m_1. Taking $m_1 = 2.00\,kg$, $m_2 = 3.00\,kg$, $m_3 = 5.00\,kg$, and $F = 20.0\,N$.

(a.) find the acceleration,

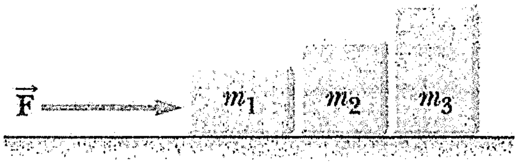
(b.) find the net force on m_1,

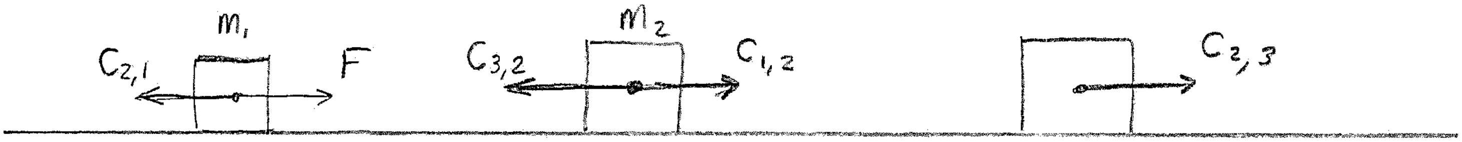
(c.) find the net force on m_2,

(d.) find the net force on m_3,

(e.) find the magnitude of the contact forces between m_1 and m_2,

(f.) find the magnitude of the contact forces between m_2 and m_3.





$C_{2,1}$ = contact force of m_2 on m_1
$C_{1,2}$ = contact force of m_1 on m_2

$\underbrace{C_{1,2} = C_{2,1}}_{\text{Newton's 3rd Law}}$

Likewise, $C_{3,2} = C_{2,3}$

For convenience we'll just write $C_{1,2} = C_{2,1} = C_{12}$ and $C_{2,3} = C_{3,2} = C_{23}$. Thus Newton's 2nd Law on masses m_1, m_2, m_3 yields

(a.)
$$\left.\begin{aligned} m_1 a &= F - C_{12} \\ m_2 a &= C_{12} - C_{23} \\ m_3 a &= C_{23} \end{aligned}\right\} \text{add these} \quad (m_1 + m_2 + m_3)a = F$$

$$\boxed{a = \frac{F}{m_1 + m_2 + m_3} = 2\,m/s^2}$$

(b.) $F_{\text{net on } m_1} = m_1 a = (2.0\,kg)(2\,m/s^2) = \boxed{4N}$

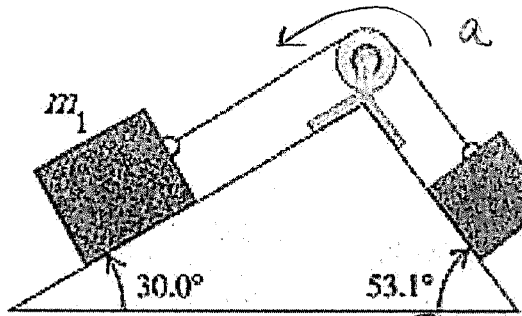
(c.) $F_{\text{net on } m_2} = m_2 a = (3.0\,kg)(2\,m/s^2) = \boxed{6N}$

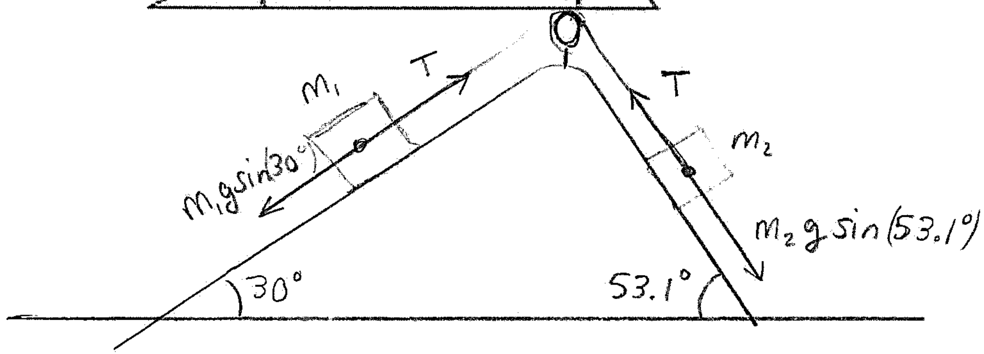
(d.) $F_{\text{net on } m_3} = m_3 a = (5.0\,kg)(2\,m/s^2) = \boxed{10N}$

(e.) $C_{12} = F - m_1 a = 20N - 4N = \boxed{16N}$

(f.) $C_{23} = m_3 a = \boxed{10N}$

Problem 34: (2pts) Two blocks connected by a cord passing over a small, frictionless pulley rest on frictionless planes. You're given $m_1 = 102\,kg$, and $m_2 = 53\,kg$. What is the magnitude of the acceleration of the blocks and what is the tension in the cord ?





$$
+\begin{pmatrix} m_1 a = m_1 g \sin 30^\circ - T \\ m_2 a = T - m_2 g \sin(53.1^\circ) \end{pmatrix}
$$

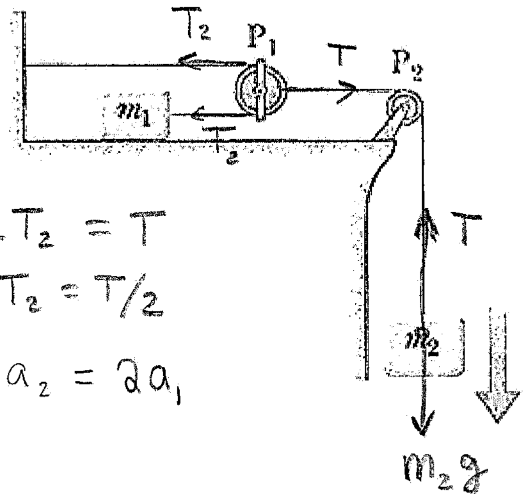
$$
(m_1 + m_2)\,a = m_1 g \sin(30^\circ) - m_2 g \sin(53.1^\circ)
$$

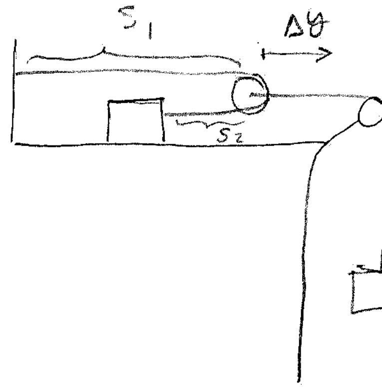
$$
\boxed{a = \left(\frac{m_1 \sin(30) - m_2 \sin(53.1)}{m_1 + m_2}\right) g = 0.5448\ m/s^2}
$$

Then,

$$
\boxed{T = m_1 g \sin(30) - m_1 a = 444.2\,N}
$$

Problem 35: (2pts) An object of mass m_1 on a frictionless horizontal table is connected to an object of mass m_2 through a very light pulley P_1 and a fixed pulley P_2 as shown in the diagram. Find the accelerations of m_1 and m_2 in terms of the masses and g.





$$2T_2 = T$$
$$T_2 = T/2$$
$$a_2 = 2a_1$$

$s_1 + s_2 = l$ ← fixed length

$$\Delta s_2 = -\Delta y$$
$$\Delta s_1 = \Delta y$$

$$m_1 a_1 = T/2$$
$$m_2 a_2 = m_2 g - T$$

Thus, using $a_2 = 2a_1$,

$$\begin{aligned} & 2m_1 a_1 = T \\ + \; & 2m_2 a_1 = m_2 g - T \\ \hline & 2(m_1 + m_2) a_1 = m_2 g \end{aligned}$$

$\Rightarrow \boxed{a_1 = \dfrac{m_2 g}{2(m_1 + m_2)}}$ acceleration of m_1

and $\boxed{a_2 = \dfrac{m_2 g}{m_1 + m_2}}$ acceleration of m_2

Problem 36: (2pts) Suppose the acceleration $a = -\beta v$ where v is velocity and $\beta > 0$ is a constant. Let v_o and x_o denote the velocity and position at $t = 0$.

(a.) Calculate the velocity as a function of position x.

(b.) Calculate the velocity as a function of time t.

Hint: for one-dimensional motion, $a = \frac{dv}{dt} = \frac{dx}{dt}\frac{dv}{dx} = v\frac{dv}{dx}$. *It follows you need to sovle* $\beta v = v\frac{dv}{dx}$ *to solve part (a.). In contrast, you need to solve* $\frac{dv}{dt} = \beta v$ *to solve part (b.).*

(a.) $$a = -\beta V = \frac{dV}{dt} = \frac{dx}{dt}\frac{dV}{dx} = V\frac{dV}{dx}$$

$$-\beta V = V\frac{dV}{dx}$$

$$\int_{V_0}^{V_f} dV = -\int_{X_0}^{X_f} \beta \, dx$$

$$V_f - V_0 = -\beta(X_f - X_0)$$

$$\boxed{V = V_0 - \beta(X - X_0)}$$

(b.) $$-\beta V = \frac{dV}{dt}$$

$$\int_{V_0}^{V_f} \frac{dV}{V} = -\int_0^{t_f} \beta \, dt$$

$$\ln|V_f| - \ln|V_0| = -\beta t_f$$

$$\ln|V_f/V_0| = -\beta t_f$$

$$\left|\frac{V_f}{V_0}\right| = \exp(-\beta t_f)$$

$$\Rightarrow \boxed{V = V_0 e^{-\beta t}}$$