Your solutions should be neat, correct and complete. Same instructions as Mission 1 apply here.

Recommended Homework from Textbook: problems:

Chapter 8 #'s 3, 13, 23, 29, 31

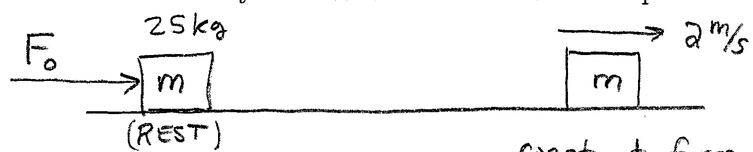
Recommended Homework from Recommended Textbook (Young & Freedman, 9th ed): Chapter 7 (potential energy and energy conservation) #'s 1, 3, 7, 11, 13, 17, 27, 31, 33, 35, 37, 41, 47, 49, 55, 59, 61, 69

Suggested Reading the following resources may be helpful:

(a.) Lectures 15, 16, 17, 18, and 19 as posted on the course website,

(b.) Chapter 8 of the required text.

Problem 49: (2pts) A 25 kg crate is pushed across the floor of a factory starting from rest and ending at a speed of $2m/s$. If a constant pushing force of 100 N was applied and the floor which is essentially frictionless then how far was required to build up the given speed.

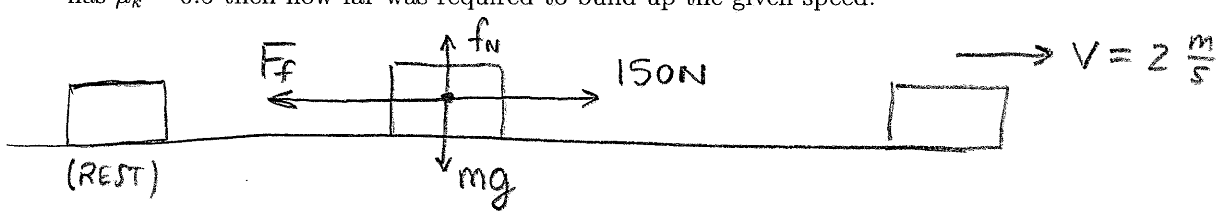


constant force in direction of the displacement Δx

$$W = F_o \Delta x = \Delta KE$$

$$\Delta x = \frac{\frac{1}{2}mv^2}{F_o} = \frac{(25\,kg)(2\,m/s)^2}{2(100N)} = \boxed{0.5\,m}$$

Problem 50: (2pts) A 25 kg crate is pushed across the floor of a factor starting from rest and ending at a speed of $2m/s$. If a constant pushing force of 150 N was applied and the floor which has $\mu_k = 0.5$ then how far was required to build up the given speed.



Then $F_f = \mu_k f_N = \mu_k mg$ so $F_{net} = 150N - \mu_k mg$

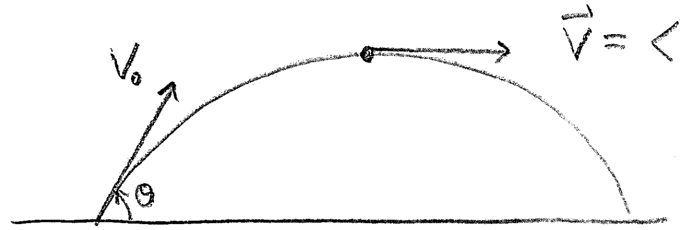
$$F_{net} = 150N - (0.5)(25kg)(9.8\,m/s^2) = 27.5\,N$$

$$W = F_{net}\Delta x = \Delta KE = \frac{1}{2}mv^2 - 0$$

$$\Delta x = \frac{\frac{1}{2}mv^2}{F_{net}} = \frac{(25kg)(2\,m/s)^2}{2(27.5N)} = \boxed{1.818\,m}$$

at top of flight

Problem 51: (2pts) A projectile is shot with a speed v_o at an angle of inclination θ such that it has 1/2 as much kinetic energy is half the initial kinetic energy. Find θ.



$$\vec{V} = \langle V_0 \cos\theta, 0 \rangle$$

$$KE_{TOP} = \frac{1}{2} KE_{BEGIN}$$

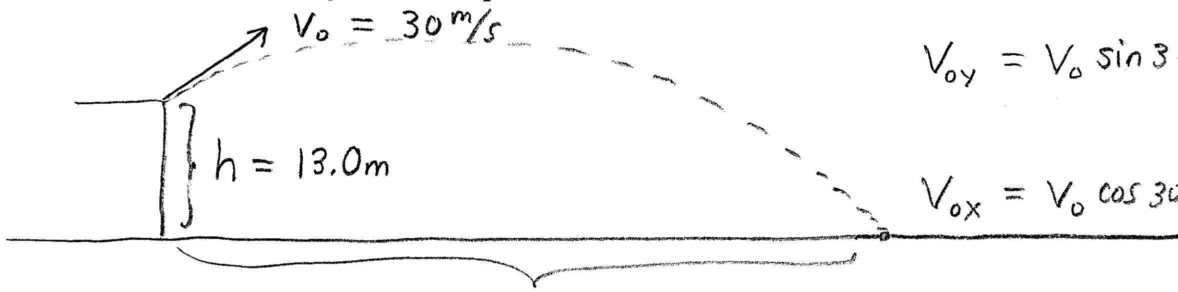
$$\frac{1}{2} m (V_0 \cos\theta)^2 = \frac{1}{2} m V_0^2$$

$$\cos^2\theta = 1$$

$$\cos\theta = \pm 1 \Rightarrow \boxed{\theta = 45^\circ} \text{ or } 135^\circ$$

also ok, don't count off for missing this one.

Problem 52: (2pts) A man stands on the roof of a 13.0 m tall building and throws a rock with a velocity of magnitude 30.0 m/s at an angle of 30° above the horizontal. You can ignore air resistance. Assuming the building is on a horizontal plane, **find how far the rock travels horizontally from the point where it was thrown.**



$$V_{oy} = V_0 \sin 30^\circ = 15\,m/s$$

$$V_{ox} = V_0 \cos 30^\circ = 25.98\,m/s$$

$$\Delta x = (V_0 \cos 30^\circ) t$$

$$mgh + \frac{1}{2} m V_0^2 = \frac{1}{2} m V_f^2 = \frac{1}{2} m V_{fx}^2 + \frac{1}{2} m V_{fy}^2$$

$$V_{fy} = -\sqrt{2gh + V_0^2 - V_0^2 \cos^2\theta}$$

$$= -\sqrt{2gh + V_0^2 \sin^2\theta}$$

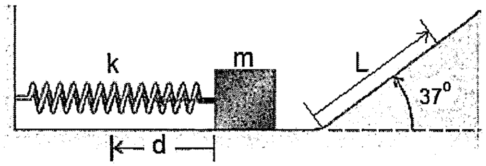
$$= -\sqrt{2(9.8\,m/s^2)(13.0\,m) + (30\,m/s)^2 (0.5)^2}$$

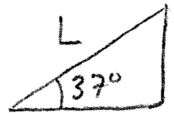
$$= -21.904\,m/s$$

But, $V_{fy} = V_{oy} - gt \Rightarrow t = \dfrac{V_{fy} - V_{oy}}{-g} = \dfrac{(-21.904 - 15)\,m/s}{-9.8\,m/s^2}$

$t = 3.766\,s \quad \therefore \quad \Delta x = (25.98\,m/s)(3.766\,s) = \boxed{97.84\,m}$

Problem 53: (2pts) A $m = 3.00\ kg$ block is pushed against a spring with negligible mass and force constant $k = 220\ N/m$, compressing it $d = 1.300\ m$. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope 37.0^o. Find the maximum length L the block slides up the incline. (see diagram for L)





$h = L \sin(37°)$

Use conservation of energy,

$$\frac{1}{2}kd^2 = \frac{1}{2}mv_0^2 = mgh = mgL\sin(37°)$$

$$L = \frac{kd^2}{2mg\sin(37°)}$$

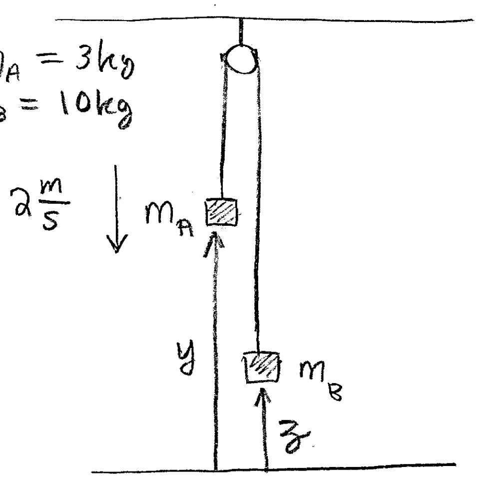
$$= \frac{(220\ N/m)(1.3 m)^2}{2(3kg)(9.8\ m/s^2)(\sin 37°)}$$

$$= \boxed{10.51\ m}$$

Problem 54: (2pts) A 3 kg and a 10 kg mass are hung on ends of a string which is placed over an essentially massless, frictionless pulley. Suppose the 3 kg mass initially is given a speed of 2 m/s downward. How far down does the mass go before it begins to travel upward ?

$KE_2 = 0$

$m_A = 3kg$
$m_B = 10kg$



$$m_A g y_1 + m_B g z_1 + \frac{1}{2}(m_A + m_B)V_1^2 = \underbrace{m_A g y_2 + m_B g z_2}_{KE_2 = 0} \quad *$$

$$z_2 - z_1 = -(y_2 - y_1)$$

$$\Delta z = -\Delta y$$

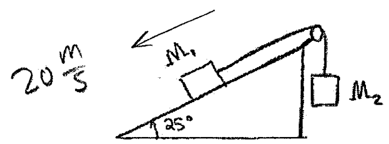
m_A goes down then m_B goes up

Rearranging $*$, $\quad m_A g(y_2 - y_1) + m_B g(z_2 - z_1) = \frac{1}{2}(m_A + m_B)V_1^2$

$$m_A g\Delta y + m_B g\Delta z = (m_A - m_B)g\Delta y = \frac{1}{2}(m_A + m_B)V_1^2$$

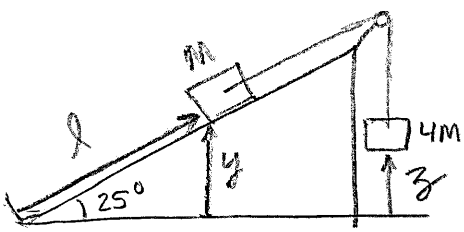
$$|\Delta y| = \left|\frac{(m_A + m_B)V_1^2}{2(m_A - m_B)g}\right| = \boxed{0.3790\ m} \quad (\text{down})$$

Problem 55: (2pts) A mass $M_1 = M$ is given velocity 20 m/s down an inclined plane as shown below. Suppose the plane is essentially frictionless and the mass is connected to a second mass $M_2 = 4M$ by a string over a frictionless, massless pulley. How far does the mass slide down the plane ?



$M_1 = M$ & $M_2 = 4M$ both have $V_0 = 20\,m/s$ since they're joined by the string.

• Conserve energy, final KE = 0



$$\frac{M}{2}V_0^2 + \frac{4M}{2}V_0^2 + Mgy_1 + 4Mgz_1 = Mgy_2 + 4Mgz_2$$

$$5V_0^2 + 2gy_1 + 8gz_1 = 2gy_2 + 8gz_2$$

$$5V_0^2 = 2g(y_2 - y_1) + 8g(z_2 - z_1)$$

$$5V_0^2 = 2g\,\Delta y + 8g\,\Delta z$$

$$5V_0^2 = 2g\sin\theta\,\Delta\ell - 8g\,\Delta\ell$$

$$|\Delta\ell| = \left|\frac{5V_0^2}{g(2\sin\theta - 8)}\right| = \boxed{28.52\,m}$$

$$\frac{\Delta y}{\Delta\ell} = \sin(25°)$$

$$\Delta y = (\sin\theta)\Delta\ell$$

$$\Delta\ell = -\Delta z$$

Problem 56: (2pts) A force parallel to the x-axis acts on a particle moving along the x-axis. This force produces a potential energy $U(x) = \alpha x^3$ where $\alpha = 2.5\,J/m^3$. What is the force (magnitude and direction) when the particle is at $x = 1.60\,m$?

$$F = -\frac{dU}{dx} = -\frac{d}{dx}(\alpha x^3) = -3\alpha x^2$$

$$F(x) = -3\alpha x^2$$

$$F(1.6m) = -3\left(2.5\,\frac{Nm}{m^3}\right)(1.6m)^2$$

$$F(1.6m) = -19.2N$$

$|F(1.6m)| = 19.2\,N$ (magnitude)
direction is in the negative x-direction (leftward)

Problem 57: (2pts) Suppose a projectile is propelled by explosive gases in the barrel of an experimental long gun according to the force $F(x) = \alpha x - \beta x^3$ where the projectile begins at rest where $x = 0$ and it exits the barrel at $x = 10\,m$. Given that the mass of the projectile is 30 grams and $\alpha = 3000\,N/m$ and $\beta = 20\,N/m^3$ find the maximum range of the gun assuming a level testing ground on Earth.

$$W_{\text{bullet in barrel}} = \int_0^{10} (\alpha x - \beta x^3)\,dx$$

$$= \left(\frac{\alpha}{2}x^2 - \frac{\beta}{4}x^4\right)\Big|_0^{10}$$

$$= \frac{1}{2}(3000)(10)^2 - \frac{1}{4}(20)(10)^4$$

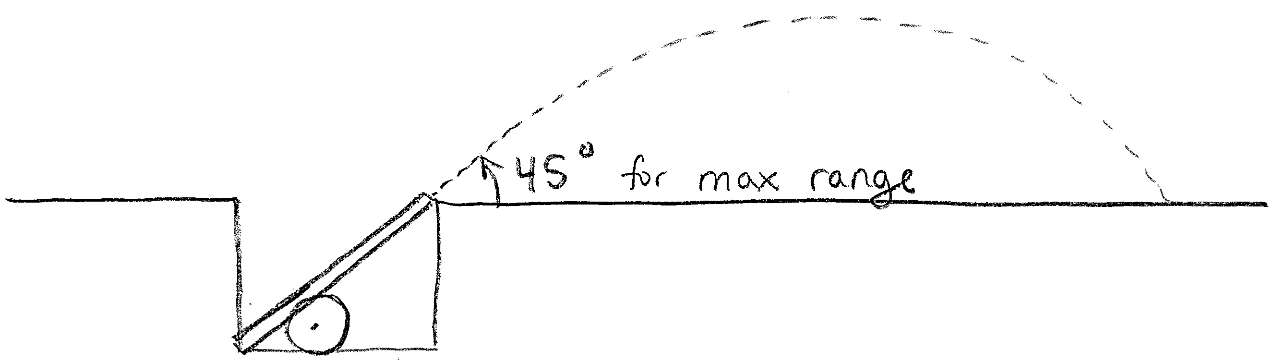
$$= 150{,}000 - 50{,}000$$

$$= 100{,}000 \quad \left(\text{in } J = N \cdot m = \frac{kg\,m^2}{s^2}\right)$$

<u>Work Energy Th^m</u>

$$200\,000\,J = \frac{1}{2}mV_0^2$$

$$V_0 = \sqrt{\frac{2(100000\,J)}{0.030\,kg}} = \underline{2581.99\,m/s}$$



$$R = \frac{V_0^2 \sin(2\theta)}{g} = \frac{V_0^2}{g} \quad \underline{\text{for } \theta = 45°}$$

$$R_{max} = \frac{(2582\,m/s)^2}{9.8\,m/s^2} = 680{,}272\,m$$

$$\boxed{R_{max} = 6803\,km = 6.803 \times 10^6\,m}$$

Problem 58: (2pts) The potential energy of two atoms in a diatomic molecule is approximated by $U(r) = a/r^{12} - b/r^6$ where r is the spacing between atoms and $a, b > 0$ are constants.

(a.) find the force $F(r)$ on one atom as a function of r,

(b.) sketch the graph of both $F(r)$ and $U(r)$ verses r

(c.) find the equilbrium distance between the atoms, is this a stable equilbrium ?

(d.) suppose the distance between the two atoms is equal to equilbrium distance found in part (b.). What minimum energy must be added to the molecule to dissociate it, that is, to separate the two atoms to an infinite distance apart ?

(a.) $F = -\frac{dU}{dr} = \frac{12a}{r^{13}} - \frac{6b}{r^7} = \boxed{\frac{1}{r^7}\left(\frac{12a}{r^6} - 6b\right)}$

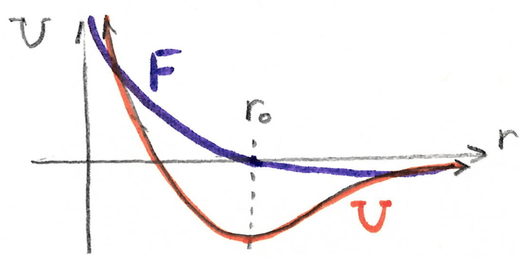
(b.) Assuming $a > 0$, $U(r) = \frac{1}{r^6}\left(\frac{a}{r^6} - b\right)$

U has min/max where $\frac{dU}{dx} = -F = 0 \Rightarrow \underbrace{\frac{12a}{r^6} - 6b = 0}$

We find critical radius of $r_0 = \left(\frac{2a}{b}\right)^{1/6}$

$\frac{12a}{r^6} = 6b$

$\frac{12a}{6b} = r^6$



(c.) equilbrium distance is $r_0 = \left(\frac{2a}{b}\right)^{1/6}$, it is <u>stable</u>.

(d.) $|U(r_0)| = \left|\frac{a}{r_0^{12}} - \frac{b}{r_0^6}\right| = \left|\frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)}\right| = \left|\frac{b^2 a}{4a^2} - \frac{b^2}{2a}\right| = \boxed{\frac{b^2}{4a}}$

Problem 59: (2pts) For the molecule O_2, the equilbrium distance is $1.21 \times 10^{-10} m$ and the dissociation energy is $8.27 \times 10^{-19} J$ per molecule. Find the values of a and b as defined in the previous problem.

$1.21 \times 10^{-10} = r_0 = \left(\frac{2a}{b}\right)^{1/6} \longrightarrow \frac{2a}{b} = (1.21 \times 10^{-10})^6$

$a = \frac{(1.21 \times 10^{-10})^6 b}{2}$

$\underbrace{8.27 \times 10^{-19} = U(r_0) = \frac{b^2}{4a}}$

$a = \frac{b^2}{4(8.27 \times 10^{-19})} = \frac{(1.21 \times 10^{-10})^6 b}{2}$

$b = (1.21 \times 10^{-10})^6 (2)(8.27 \times 10^{-19}) \Rightarrow \boxed{b = 5.191 \times 10^{-78}}$

$a = \frac{b^2}{4(8.27 \times 10^{-19})} = 0.8146 \times 10^{2(-78)+19} \Rightarrow \boxed{a = 8.146 \times 10^{-138}}$

Problem 60: (2pts) Suppose we shoot an arrow at angle β above the horizontal at the base of an inclined plane with angle of inclination α. What angle β maximizes the range R ?