Your solutions should be neat, correct and complete. Same instructions as Mission 1 apply here.

Recommended Homework from Textbook: problems:

9.3, 9.7, 9.13, 9.22, 9.35, 9.39, 9.45, 9.78, 9.84, 9.99

10.1, 10.4, 10.11, 10.17, 10.34, 10.39, 10.40, 10.43, 10.50, 10.64, 10.67, 10.79, 10.97,

13.5, 13.14, 13.16, 13.22, 13.29, 13.32, 13.37, 13.43, 13.52, 13.62, 13.75.

I also reccommend you work on understanding whatever details of lecture seem mysterious at first.

Required Reading 6 [1pt] Your signature below indicates you have read:

(a.) I read Lectures 26, 27, 28, 29, 30, 31, 32 and 33 by Cook as announced in Blackboard: ________________.

(b.) I read Chapter 9, 10 and 13 of the required text: ________________.

Problem 51 [3pts] A yo-yo has 300 J of energy in the form of rotational kinetic energy. The yo-yo also has an angular momentum of $L = 20\, m^2 kg/s$. What is the moment of intertia of the yo-yo ?

$$\left.\begin{array}{l} KE = \frac{1}{2} I \omega^2 \Rightarrow \omega^2 = \frac{2K}{I} \\ L = I\omega \Rightarrow \omega^2 = \frac{L^2}{I^2} \end{array}\right\} \quad \frac{2K}{I} = \frac{L^2}{I^2}$$

Thus, $\frac{I^2}{I} = \frac{L^2}{2K} \Rightarrow I = \frac{L^2}{2K}$

$$I = \frac{(20\ m^2 kg/s)^2}{2(300\ Nm)} = \boxed{0.667\ kg\ m^2}$$

$$\frac{m^4 kg^2}{\cancel{s^2}\, \frac{kg\, m^2}{\cancel{s^2}}} \neq kg\, m^2$$

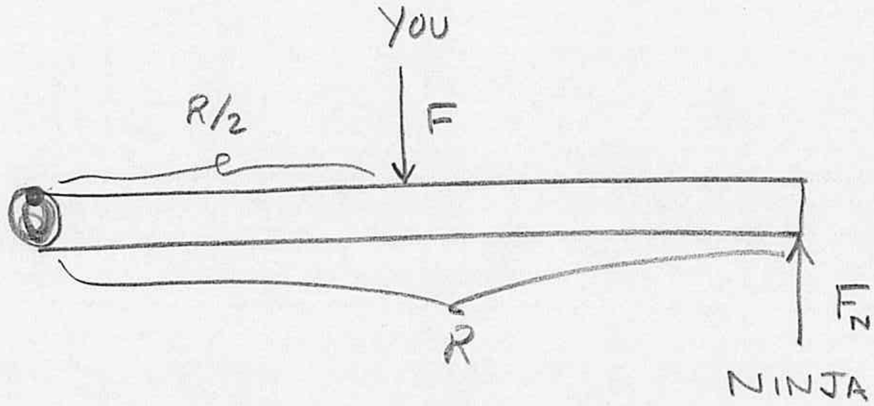
Problem 52 [3pts] A force $\vec{\mathbf{F}} = (3.0\,N)(3\hat{\mathbf{x}} - 2\hat{\mathbf{z}})$ is applied to a point mass $M = 2.00\,kg$ which has initial velocity $\vec{\mathbf{v}} = (30\,m/s)(\hat{\mathbf{y}} + \hat{\mathbf{z}})$ at the point $(1,2,3)m$. Find the torque on M and the angular momentum of M with respect to the origin.

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$= \langle 1,2,3\rangle m \times (3.0N)\langle 3,0,-2\rangle$$
$$= 3J\left[\hat{x}\times(3\cancel{\hat{x}}^{\,0} - 2\hat{z}) + 2\hat{y}\times(3\hat{x} - 2\hat{z}) + 3\hat{z}\times(3\hat{x} - \cancel{2\hat{z}}^{\,0})\right]$$
$$= 3J\left[2\hat{y} - 6\hat{z} - 4\hat{x} + 9\hat{y}\right]$$
$$= \boxed{3J\langle -4, 11, -6\rangle} \Leftarrow \text{torque.}$$

aka. $\underline{\langle -12J, 33J, -18J\rangle = \vec{\tau}}$

$$\vec{L} = \vec{r} \times \vec{p}$$
$$= m\,\vec{r} \times \vec{v}$$
$$= (2.00kg)\left(\langle 1,2,3\rangle(m)\right) \times \left[30\tfrac{m}{s}(\hat{y} + \hat{z})\right]$$
$$= (60J)\left(\langle 1,2,3\rangle \times \langle 0,1,1\rangle\right)$$
$$= (60J)\langle 2-3, -1, 1\rangle$$
$$= 60J\langle -1,-1,1\rangle$$
$$= \boxed{\langle -60J, -60J, 60J\rangle = \vec{L}}$$

Problem 53 [3pts] You push the edge of a door of large square door with side-length 2.00 m at the middle of the door. A mischevious genin-level ninja who just learned about mechanical advantage pushes at the edge of the door and stops your push with a smaller force. If you push with force F then what force did the ninja stop you ?

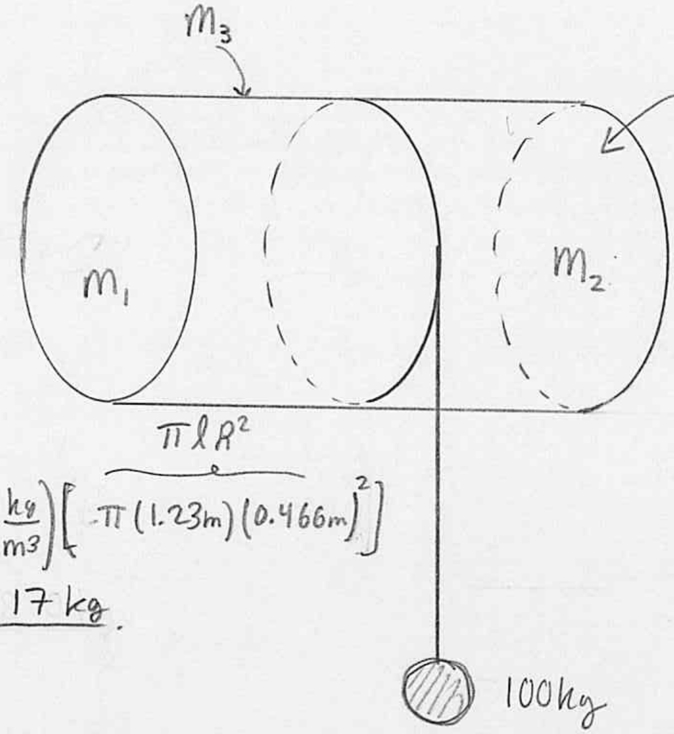


$$\tau_{net} = -\frac{R}{2}F + RF_N = 0$$

$$\Rightarrow \boxed{F_N = F/2}$$

(I didn't need to give you R = 2.00 m)

Problem 54 [3pts] A mass of 100 kg hangs via a very thin wire (with small mass) of the edge of a cylindrical barrel filled with cheese of density $30kg/m^3$. The barrel itself has a mass of 20 kg which includes the caps and the sides. The cylinder has length of $l = 1.23\,m$ and a radius of $R = 0.466\,m$. If the barrel rotates on an essentially frictionless axel then how far will the mass fall in the first second it is released from rest ? How much rotational energy and how much angular momentum will be given to the cheese barrel at that time ?



For barrel itself,

$$\sigma = \frac{dm}{dA} = \frac{20kg}{\text{total area}}$$

$$\sigma = \frac{20kg}{2\pi R^2 + 2\pi R l}$$

$$\sigma = 4.028 \frac{kg}{m^2}$$

$$m_{cheese} = \left(30\frac{kg}{m^3}\right)\Big[\overbrace{\pi(1.23m)(0.466m)^2}^{\pi l R^2}\Big]$$

$$= \underline{25.17\,kg}.$$

$$m_1 = \pi R^2 \sigma = 2.748\,kg$$

$$m_2 = \pi R^2 \sigma = 2.748\,kg$$

$$m_3 = 2\pi R l \sigma = 14.51\,kg$$

The moment of inertia for disk is $\frac{1}{2}mR^2$,

$$I = I_1 + I_2 + I_3 + I_{cheese}$$

$$= \frac{1}{2}m_1R^2 + \frac{1}{2}m_2R^2 + m_3R^2 + \frac{1}{2}m_{cheese}R^2$$

$$= (m_1 + m_3)R^2 + \frac{1}{2}m_{chease}R^2$$

$$= \left(m_1 + m_3 + \frac{1}{2}m_{chease}\right)R^2 = \underline{6.481\,kg\,m^2 = I_{total}}.$$

Thus, $\tau = I\alpha \Rightarrow (100kg)(9.8\frac{m}{s^2})(0.466m) = (6.481)kg\,m^2\alpha$

Hence $\underline{\alpha = 70.46\,rad/s^2}. \Rightarrow \underline{a = R\alpha = 32.84\,m/s^2}.$

$$y = \cancel{y_0}^{\,0} + \cancel{v_0}^{\,0}t + \frac{1}{2}at^2 \Rightarrow y(1) = \frac{1}{2}(32.84\frac{m}{s^2})(1.0s)^2 = \boxed{16.42m = \Delta y}$$

$$\omega = \omega_0 + \alpha t \Rightarrow \omega_f = (70.46\frac{rad}{s^2})(1.0s) = 70.46\frac{rad}{s}$$

$$L_f = I\omega_f = (6.481\,kgm^2)(70.46\frac{rad}{s}) = \boxed{456.7\frac{kg\,m^2}{s} = L_f}$$

$$KE_f = \frac{L_f^2}{2I} = \boxed{16.09\,kJ}$$

Problem 55 [3pts] Problem 13.85 (uniform earth PE, fall to center)

If the earth had $\rho = \frac{dM}{dV}$ = constant then it can be shown that gravity decreases linearly to zero from its max. value on the surface, in particular,

attractive !

$$F_g = \frac{-G M_E m r}{R_E^3}$$

[Note: if $r \geq R_E$ then, $F_g = -\frac{G m M_E}{r^2}$ for]

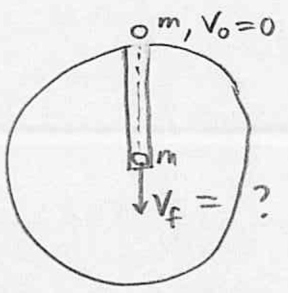
(a.) find $U(r)$ for $0 < r < R_E$,

$$F_g = -\frac{dU}{dr} \implies \boxed{U(r) = U_o + \frac{1}{2}\frac{G M_E m}{R_E^3} r^2}$$

Assume $U(0) = 0 \implies \underline{U_o = 0}$ thus,

$$\boxed{U(r) = \frac{1}{2}\left(\frac{G M_E m}{R_E^3}\right) r^2}$$

(b.) Assuming PE that derived in (a.) is valid what is speed of mass which falls to $r = 0$?



$$U(R_E) = KE_f$$

$$\cancel{\tfrac{1}{2}}\left(\frac{G M_E \cancel{m}}{R_E^3}\right) R_E^2 = \cancel{\tfrac{1}{2}}\cancel{m} V_f^2$$

$$V_f = \sqrt{\frac{G M_E}{R_E}}$$

$$= \sqrt{\frac{(6.673 \times 10^{-11})(5.97 \times 10^{24})}{6.387 \times 10^6}} \ \frac{m}{s}$$

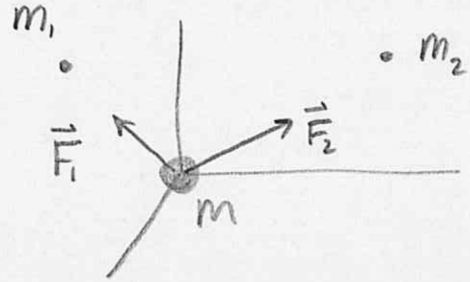
$$= \boxed{7,897.7 \ m/s}$$

Problem 56 [3pts] Let masses $m_1 = 1.0\,kg$ be placed at $(1.0\,m, 0, 3.0\,m)$ and $m_2 = 2.0\,kg$ be placed at $(-1.0\,m, 2.0\,m, 0)$. Find the net gravitational force on $M = 0.030\,kg$ placed at the origin. What is the gravitational acceleration due to m_1 and m_2 at the origin?

$m_1 = 1.0kg$ at $\vec{r}_1 = \langle 1, 0, 3 \rangle m$

$m_2 = 2.0kg$ at $\vec{r}_2 = \langle -1, 2, 0 \rangle m$

place $M = 0.03\,kg$ at $(0,0,0)$ find net gravitational force on M due to m_1 & m_2.



$$\vec{F}_1 = \frac{Gm_1M}{r_1^3}\vec{r}_1$$

$$\vec{F}_2 = \frac{Gm_2M}{r_2^3}\vec{r}_2$$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

$$= GM\left(\frac{m_1}{(\sqrt{10})^3 m^3}\langle 1, 0, 3\rangle m + \frac{m_2}{(\sqrt{5})^3 m^3}\langle -1, 2, 0\rangle m\right)$$

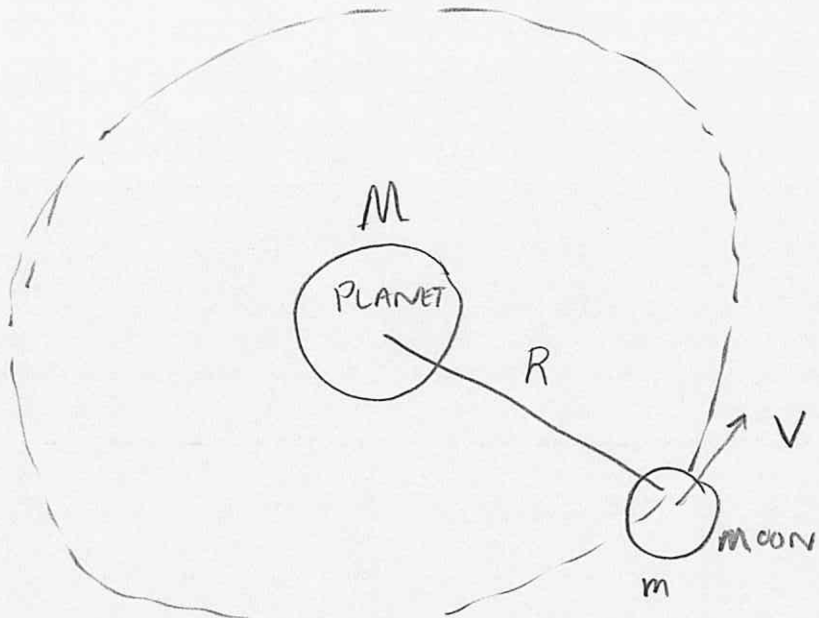
$$= \left(6.673\times10^{-11}\,\frac{Nm^2}{kg^2}\right)(0.03kg)\left[\langle 0.03162, 0, 0.09487\rangle + \langle -0.1789, 0.3578, 0\rangle\right]\frac{kg}{m^2}$$

$$= \left(2.0019\times10^{-12}\,\frac{Nm^2}{kg}\right)\langle -0.1473, 0.3578, 0.09487\rangle\frac{kg}{m^2}$$

$$= \boxed{\langle -0.2949\times10^{-12}N,\ 0.7163\times10^{-12}N,\ 0.19\times10^{-12}N\rangle}$$

$$\vec{a} = \frac{1}{M}\vec{F}_{net} = \boxed{\langle -9.83,\ 23.88,\ 6.333\rangle\times10^{-12}\,\frac{m}{s^2}}$$

Problem 57 [3pts] A planet has mass $M = 3.54 \times 10^{27}\,kg$. A moon orbits the planet in a circular orbit of radius $R = 2.0 \times 10^8\,m$. What is the period of the moon's orbit?



net-force must follow $\frac{mv^2}{R}$ for circular orbit.

$$\underbrace{\frac{G\cancel{m}M}{R^2}} = \frac{\cancel{m}v^2}{R} = \frac{1}{R}\left(\frac{2\pi R}{T}\right)^2$$

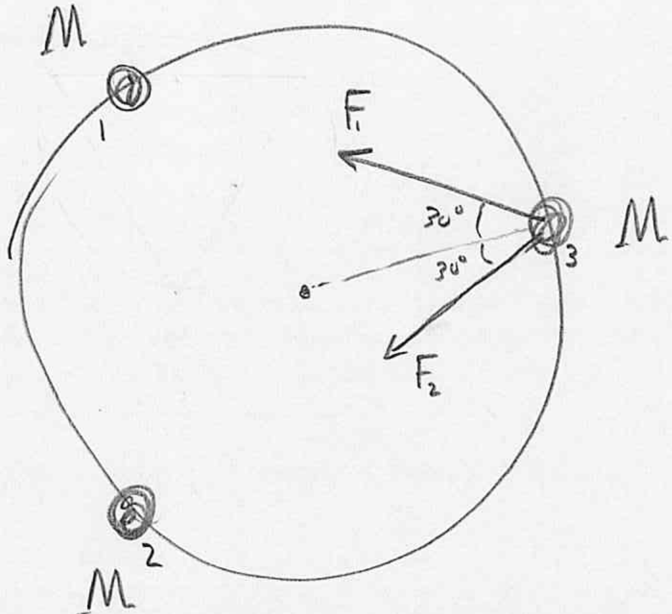
$$\frac{GM}{R} = \frac{4\pi^2 R^2}{T^2}$$

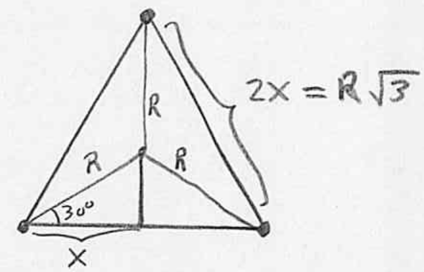
$$T = \sqrt{\frac{4\pi^2 R^3}{GM}} = 2\pi\sqrt{\frac{R^3}{GM}}$$

$$T = 2\pi\sqrt{\frac{(2\times10^8\,m)^3}{(6.673\times10^{-11})\frac{N\,m^2}{kg^2}\;3.54\times10^{27}\,kg}}$$

$$\boxed{T = 3.656\times10^4\,s}$$

Problem 58 [3pts] Three planets of identical mass M orbit in a circular orbit of radius R. The planets are symmetrically placed. Find the speed of their orbit.





$$X = R\cos 30° = \frac{R\sqrt{3}}{2}$$

Net force on 3 is radially inword,

$$F_{net} = (\cos 30°)\,F_1 + (\cos 30°)\,F_2$$

$$= 2\cos(30°)\,\frac{GMM}{(R\sqrt{3})^2}$$

Thus, noting motion of 3 is given to be circular,

$$\frac{\cancel{M}v^2}{R} = \sqrt{3}\,\frac{G\cancel{M}M}{R^2(\sqrt{3})^2} = \frac{G\cancel{M}M}{R^2\sqrt{3}}$$

$$\boxed{v = \sqrt{\frac{GM}{R\sqrt{3}}}}$$

PROBLEM 59

10.81. **IDENTIFY:** Apply conservation of energy to the motion of the boulder.

SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ and $v = R\omega$ when there is rolling without slipping. $I = \frac{2}{5}mR^2$.

EXECUTE: Break into two parts, the rough and smooth sections.

Rough: $mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$. $v^2 = \frac{10}{7}gh_1$.

Smooth: Rotational kinetic energy does not change. $mgh_2 + \frac{1}{2}mv^2 + K_{\text{rot}} = \frac{1}{2}mv_{\text{Bottom}}^2 + K_{\text{rot}}$.

$gh_2 + \frac{1}{2}\left(\frac{10}{7}gh_1\right) = \frac{1}{2}v_{\text{Bottom}}^2$. $v_{\text{Bottom}} = \sqrt{\frac{10}{7}gh_1 + 2gh_2} = \sqrt{\frac{10}{7}(9.80\text{ m/s}^2)(25\text{ m}) + 2(9.80\text{ m/s}^2)(25\text{ m})} = 29.0\text{ m/s}$.

EVALUATE: If all the hill was rough enough to cause rolling without slipping,

$v_{\text{Bottom}} = \sqrt{\frac{10}{7}g(50\text{ m})} = 26.5\text{ m/s}$. A smaller fraction of the initial gravitational potential energy goes into translational kinetic energy of the center of mass than if part of the hill is smooth. If the entire hill is smooth and the boulder slides without slipping, $v_{\text{Bottom}} = \sqrt{2g(50\text{ m})} = 31.3\text{ m/s}$. In this case all the initial gravitational potential energy goes into the kinetic energy of the translational motion.

Problem 60 [3pts] Problem 13.79 (mars rocket orbital modification)

13.79. **IDENTIFY** and **SET UP:** Apply conservation of energy (Eq. (7.13)) and solve for W_{other}. Only $r = h + R_E$ is given, so use Eq. (13.10) to relate r and v.

EXECUTE: $K_1 + U_1 + W_{other} = K_2 + U_2$

$U_1 = -Gm_M m/r_1$, where m_M is the mass of Mars and $r_1 = R_M + h$, where R_M is the radius of Mars and $h = 2000\times10^3$ m.

$$U_1 = -(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\ \text{kg})(5000\ \text{kg})}{3.40\times10^6\ \text{m} + 2000\times10^3\ \text{m}} = -3.9667\times10^{10}\ \text{J}$$

$U_2 = -Gm_M m/r_2$, where r_2 is the new orbit radius.

$$U_2 = -(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\ \text{kg})(5000\ \text{kg})}{3.40\times10^6\ \text{m} + 4000\times10^3\ \text{m}} = -2.8950\times10^{10}\ \text{J}$$

For a circular orbit $v = \sqrt{Gm_M/r}$ (Eq. (13.10)), with the mass of Mars rather than the mass of the earth).

Using this gives $K = \frac{1}{2}mv^2 = \frac{1}{2}m(Gm_M/r) = \frac{1}{2}Gm_M m/r$, so $K = -\frac{1}{2}U$.

$K_1 = -\frac{1}{2}U_1 = +1.9833\times10^{10}$ J and $K_2 = -\frac{1}{2}U_2 = +1.4475\times10^{10}$ J

Then $K_1 + U_1 + W_{other} = K_2 + U_2$ gives

$W_{other} = (K_2 - K_1) + (U_2 - U) = (1.4475\times10^{10}\ \text{J} - 1.9833\times10^{10}\text{J}) + (+3.9667\times10^{10}\ \text{J} - 2.8950\times10^{10}\ \text{J})$

$W_{other} = -5.3580\times10^9\ \text{J} + 1.0717\times10^{10}\ \text{J} = 5.36\times10^9$ J.

EVALUATE: When the orbit radius increases the kinetic energy decreases and the gravitational potential energy increases. $K = -U/2$ so $E = K + U = -U/2$ and the total energy also increases (becomes less negative). Positive work must be done to increase the total energy of the satellite.