Your solutions should be neat, correct and complete. Same instructions as Mission 1 apply here.

Recommended Homework from Textbook: problems:

Chapter 9 #'s 3, 5, 9, 13, 15, 19, 21, 23, 26, 27, 31, 37, 45

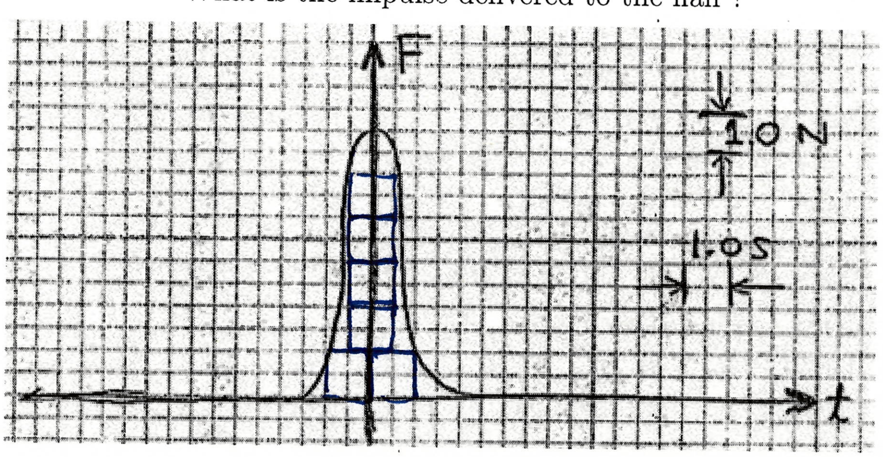
Recommended Homework from Recommended Textbook (Young & Freedman, 9th ed): Chapter 8 (momentum, impulse and collisions) #'s 3, 5, 7, 9, 11, 13, 15, 19, 21, 23, 25, 29, 31, 32, 33, 35, 39, 43, 45, 59, 61, 63, 64, 69, 71, 73, 81, 83, 91, 100

Suggested Reading the following resources may be helpful:

(a.) Lectures 21, 22, 24 as posted on the course website,

(b.) Chapter 8 of the required text.

Problem 61: (2pts) The force of a hammer hitting a nail is graphed below in a force vs. time graph. What is the impulse delivered to the nail ?



$$\Delta P = \int_{-\infty}^{\infty} F(t)\,dt$$

$$\cong \left(\frac{32 \text{ boxes}}{4 \text{ boxes}}\right)\frac{kg\,m}{s}$$

$$\cong \boxed{8\,\frac{kg\,m}{s}}$$

$$Ns = \left(\frac{kg\,m}{s^2}\right)s = \frac{kg\,m}{s} = 4 \text{ boxes}$$

Problem 62: (2pts) Suppose $m_1 = 3.0kg$ is at $\vec{\mathbf{r}}_1 = (1.0m)\langle 1,2,3\rangle$ and $m_2 = 4.0kg$ is at $\vec{\mathbf{r}}_2 = (1.0m)\langle -1,0,6\rangle$ and $m_3 = 3.0kg$ is at $\vec{\mathbf{r}}_3 = (1.0m)\langle 4,4,4\rangle$. Find the center of mass for this system of three masses.

$$\vec{R} = \frac{1}{M}\left(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3\right)$$

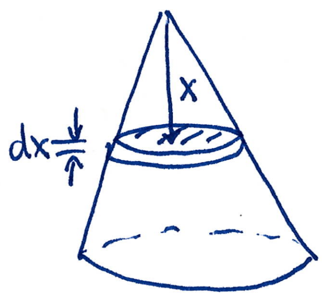
$$= \frac{1}{10kg}\left((3kg)(1.0m)\langle 1,2,3\rangle + (4kg)(1.0m)\langle -1,0,6\rangle + (3.0kg)(1.0m)\langle 4,4,4\rangle\right)$$

$$= \frac{m}{10}\left(\langle 3,6,9\rangle + \langle -4,0,24\rangle + \langle 12,12,12\rangle\right)$$

$$= \frac{m}{10}\langle 3-4+12,\ 6+12,\ 9+24+12\rangle$$

$$= \boxed{\langle 1.1m,\ 1.8m,\ 4.5m\rangle}$$

Problem 63: (2pts) Suppose the linear mass density of a cone is given by $\lambda = (3.0\,kg/m^2)x$ for $0 \leq x \leq 30\,cm$ where $x = 0$ corresponds to the tip of the cone and $x = 30\,cm$ gives the base. Find the center of mass for this distribution of mass (notice, while a cone is three-dimensional, clearly the center of mass is on the axis so we are able to treat the problem with single-variable calculus)



$$\lambda = \frac{dm}{dx}$$

$$dm = \lambda\, dx = 3\alpha x \quad \text{for } 0 \leq x \leq L$$

$$M = \int_{cone} dm = \int_0^L 3\alpha x\, dx = \frac{3\alpha L^2}{2}$$

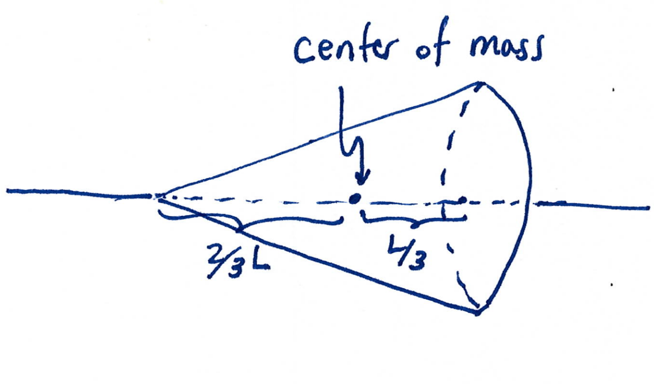
$$\bar{x} = \frac{1}{M}\int_{cone} x\, dm = \frac{2}{3\alpha L^2}\int_0^L 3\alpha x^2 dx$$

$$= \frac{2}{3\alpha L^2}\left(\alpha L^3\right)$$

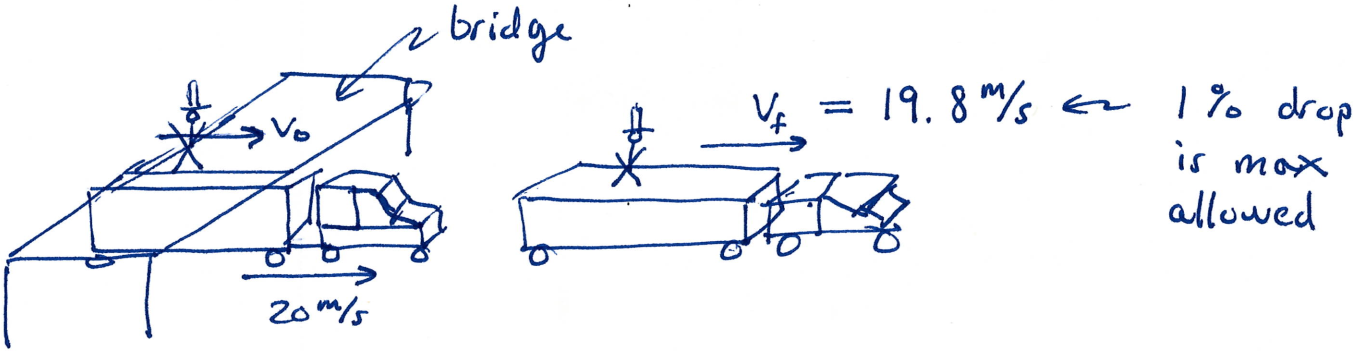
$$= \frac{2}{3} L$$

$$= \frac{2}{3}(30\,cm)$$

$$\boxed{\bar{x} = 20\,cm}$$



Problem 64: (2pts) A 3000 kg truck travels past a highway overpass at 20 m/s. A heavy ninja of mass 150kg runs from a bridge which is nearly level with the top of the truck (we can ignore vertical motion). If the truck driver will notice a change of more than 1% in the speed then what is the minimum speed the ninja must run to jump on the truck without being noticed ?



$$(150\,kg)V_o + (3000\,kg)(20\,\tfrac{m}{s}) = (3150\,kg)\,V_f$$

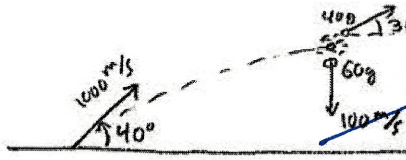
$$V_o = \frac{(3150)(19.8\,m/s) - (3000)(20\,m/s)}{150}$$

$$\boxed{V_o = 15.8\ m/s}$$

Remark: ideally the Ninja would want to match the speed of the truck. Of cource a truck which has mass of 10,000 kg would not notice the jump as easily. Notice,

$$V_{o2} = \frac{(10{,}150)(19.8\,m/s) - (10{,}000)(20\,m/s)}{150} = 6.47\ m/s$$

Problem 65: (2pts) An explosive 100 gram bullet is shot with speed 1000 m/s at an angle of 40 degrees above the horizontal. At the zenith of its trajectory it explodes into two pieces. The first piece has 60 grams of material and it falls directly downward at an initial speed 100 m/s. The second piece has 40 grams and it travels away at an initial angle of 30 degrees above the horizontal.



$737.08 \frac{m}{s}$ ← given in Announcement

(a.) How far does the 40 gram fragment travel horizontally? $\boxed{533,190\,m}$ (approximately)

(b.) What is the initial momentum of the bullet? $\boxed{\vec{P}_o = \langle 76.6\ \text{kg m/s},\ 64.3\ \text{kg m/s} \rangle}$

(c.) What is the momentum of the bullet just before it explodes? $\boxed{\vec{P}_{\text{before explode}} = \langle 76.6 \frac{kgm}{s},\ 0 \rangle}$

$$\vec{P}_{\text{before explosion}} = 0.1\,kg \left\langle \cos(40^\circ)\,1000 \frac{m}{s},\ 0 \right\rangle = \left\langle 76.6 \frac{kgm}{s},\ 0 \right\rangle$$

$$\vec{P}_{\text{after explosion}} = (0.06\,kg)\left\langle 0,\ -737.08 \frac{m}{s} \right\rangle + 0.04\,kg \left\langle V_2 \cos 30,\ V_2 \sin 30 \right\rangle$$

Conservation of momentum in the explosion gives

$$\left\langle 76.6 \frac{kgm}{s},\ 0 \right\rangle = \left\langle 0.03464\, V_2,\ (-737.08)(0.06)\frac{m}{s} + (0.04)(0.5)V_2 \right\rangle kg$$

$$\Rightarrow \quad V_2 = \frac{76.6}{0.03464}\ \frac{m}{s} = 2211.3\ m/s$$

At moment of explosion the bullet is at $\frac{1}{2}(\text{Range}) = X$ and y = max height. We know $X = \frac{1}{2}\left(\frac{V_o^2 \sin(2\theta)}{g}\right) = 50,245\,m$

and $y = \dfrac{V_o^2 \sin(\theta)}{2g} = 32,795\,m$. Then

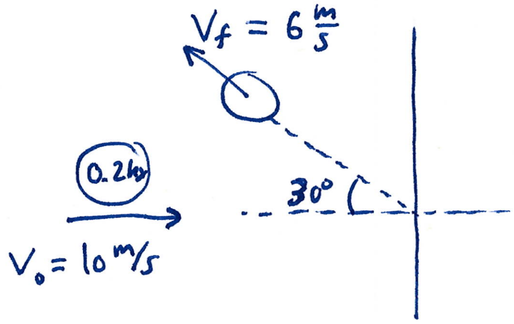
$$\Delta X = 50,245\,m + (V_2 \cos 30)t$$
$$y = 32,795\,m + (V_2 \sin 30)t - \frac{1}{2}gt^2$$

We face $0 = 32795 + (1105.65)t - 4.9t^2$ to find t from zenith until it hits ground. This yields $\underline{t = 252.18\,s}$.

Thus $\Delta X = 50,245\,m + (2211.3 \frac{m}{s}) \cos(30)(252.18\,s) = \underline{533,190\,m}$.

$$\vec{P}_o = m\vec{V}_o = (0.1\,kg)(1000 \tfrac{m}{s})\langle \cos 40,\ \sin 40 \rangle = \left\langle 76.6 \frac{kgm}{s},\ 64.3 \frac{kgm}{s} \right\rangle$$

Problem 66: (2pts) A rubber ball bounces off a vertical brick wall. If the 0.2 kg ball hits the wall horizontally at 10 m/s but bounces off at 6 m/s and an angle of 30 degrees above the horizontal then what was the impulse delivered to the ball by the wall? If the bouncing happened over a duration of 0.01 s then what was the magnitude of the force of the wall on the ball? (assume the force was constant)

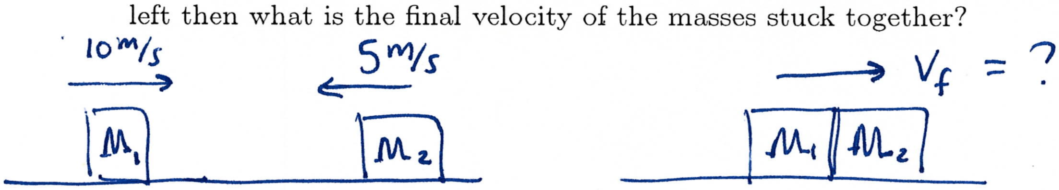


$$\Delta\vec{P} = \vec{P}_f - \vec{P}_o$$
$$= m\vec{V}_f - m\vec{V}_o$$
$$= m(\vec{V}_f - \vec{V}_o)$$
$$= (0.2kg)\left(6\tfrac{m}{s}\langle -\cos 30, \sin 30\rangle - 10\tfrac{m}{s}\langle 1, 0\rangle\right)$$
$$= (0.2kg)\langle -15.2\tfrac{m}{s}, 3\tfrac{m}{s}\rangle$$

$$\vec{F}_{avg} = \frac{\Delta\vec{P}}{\Delta t} = \frac{0.2kg}{0.01s}\langle -15.2\tfrac{m}{s}, 3\tfrac{m}{s}\rangle = \langle -304N, 60N\rangle$$

$$\|\vec{F}_{avg}\| = \sqrt{(-304N)^2 + (60N)^2} = \boxed{309.9\,N}$$

Problem 67: (2pts) Two masses collide in a purely inelastic head-on collision. If $M_1 = 2\,kg$ has an initial speed of 10 m/s to the right and $M_2 = 3\,kg$ has an initial speed of 5 m/s to the left then what is the final velocity of the masses stuck together?



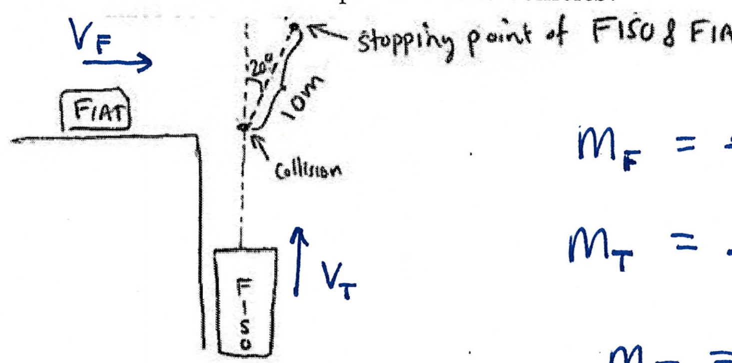
$$(2kg)(10\tfrac{m}{s}) - (3kg)(5\tfrac{m}{s}) = (5kg)V_f$$

$$V_f = \left(\frac{20-15}{5}\right)\frac{m}{s}$$

$$\therefore \boxed{V_f = 1\,\frac{m}{s}}$$

Remark: this problem is easy.

Problem 68: (2pts) A Fiat with a weight of 5000 N collides with a loaded F-150 weighing 25,000 N. Suppose the Fiat gets stuck in the grill of the pick-up truck and the combined mass skids to a stop in 10 m over a road with coefficient of kinetic friction 0.4. Furthermore, suppose the squished Fiat and F-150 travel a straight path deflected 20 degrees from the original direction of the F-150. If the collision happens at perpendicular intersection of two roads then what are the initial speeds of the vehicles?



$$M_F = \frac{5000\,N}{9.8\,m/s^2} = 510.20\ kg$$

$$M_T = \frac{25000}{9.8\,m/s^2} = 2551.02\ kg$$

$$M_T = 5M_F$$

Conserve momentum at collision, speed of $M_F + M_T$ after collision at start of the skidding

$$m_F \langle V_F, 0\rangle + m_T\langle 0, V_T\rangle = (M_F + M_T)\, V_0 \langle \sin 20, \cos 20\rangle$$

$$M_F \langle V_F, 0\rangle + 5M_F\langle 0, V_T\rangle = 6M_F V_0 \langle \sin 20, \cos 20\rangle$$

$$\langle V_F, 5V_T\rangle = \langle 6V_0 \sin 20,\ 6V_0\cos 20\rangle$$

Thus,

$$V_F = 6V_0 \sin 20 \quad \text{and} \quad V_T = \frac{6V_0}{5}\cos 20 \quad *$$

It remains to find V_0, Observe that friction does work to dissapate the $KE = \frac{1}{2}mV_0^2 = \frac{1}{2}(6M_F)V_0^2$

$$\frac{1}{2}(6M_F)V_0^2 = \mu f_N d = \mu(6M_F g)(10m)$$

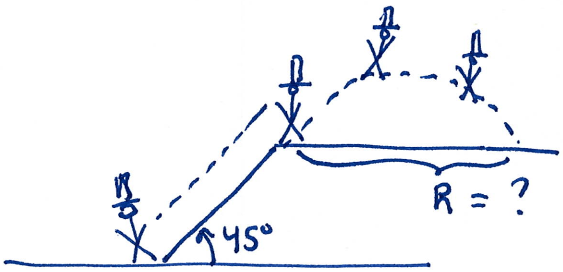
$$V_0 = \sqrt{2(0.4)(9.8\,m/s^2)(10m)} = 8.854\ m/s$$

Returning to * we find

$$\boxed{V_F = V_{FIAT} = 18.17\ m/s}$$

$$\boxed{V_T = V_{F150} = 9.98\ m/s}$$

Problem 69: (2pts) Suppose you shoot a 100 kg stunt man wearing a Kevlar vest with a 10 gram bullet at a speed of 340 m/s. If the bullet sticks then what velocity should the man be given as a result of being shot. Assuming the best possible trajectory for flight what is the maximum distance this bullet could throw him?

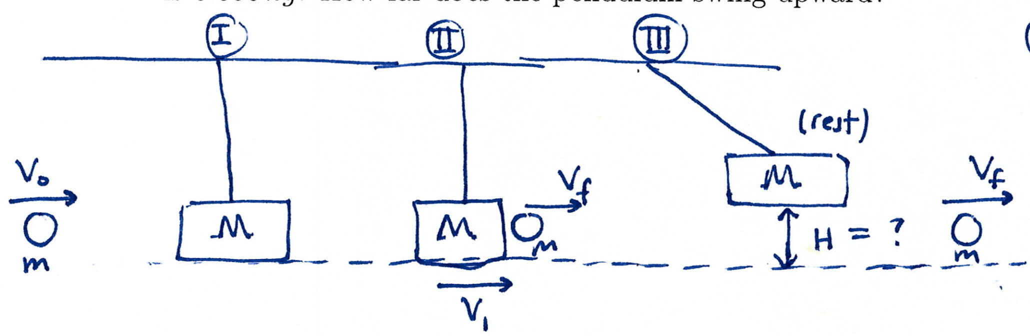


$$(0.01\,kg)(340\,\tfrac{m}{s}) = (100.01\,kg)\,V_o \Rightarrow \boxed{V_o = 0.034\ m/s}$$

$$R_{max} = \frac{V_o^2 \sin(90)}{g} = \boxed{1.179 \times 10^{-4}\,m = 0.0001179\,m}$$

Remark: if $M_{bullet} = 500$ grams then $V_o = 1.69$ m/s then $R_{max} = 0.292\,m$.

Problem 70: (2pts) A bullet is shot through a clay pendulum bob. In the process of the bullets travel through the pendulum bob it loses half of its kinetic energy. The mass of the pendulum is 0.050 kg. How far does the pendulum swing upward?



I: before hit

II: just as bullet exits the bob

III: bob gets to top of swing where KE = 0

$$mV_o = MV_1 + mV_f \quad \& \quad \underbrace{\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}\left(\tfrac{1}{2}mv_o^2\right)}_{V_o^2 = 2V_f^2} \quad \therefore \quad V_f = \frac{V_o}{\sqrt{2}}$$

$$V_1 = \frac{mV_o - mV_f}{M}$$

$$V_1 = \frac{m(1 - 1/\sqrt{2})V_o}{M} \qquad H = \frac{m^2(3 - 2\sqrt{2})V_o^2}{4M^2 g}$$

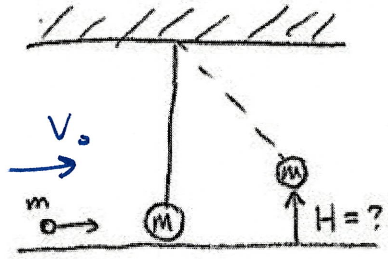
arithmetic

Conserve energy from II to III

$$\tfrac{1}{2}MV_1^2 = MgH$$

$$\therefore \quad H = \frac{V_1^2}{2g} \Rightarrow \boxed{H = \frac{m^2(1 - 1/\sqrt{2})^2 V_o^2}{2M^2 g}}$$

Problem 71: (2pts) Suppose a bullet of mass m collides with a pendulum of mass M. If the pendulum swings to a height H then what was the initial speed of the bullet given that:



(a.) the bullet stuck to the pendulum

(b.) the collision was elastic

(a.) $mV_o = (M+m)V_1 \quad \therefore \quad V_1 = \frac{mV_o}{M+m}$ speed of bob just after impact by m.

$$\frac{1}{2}(M+m)V_1^2 = (M+m)gH$$

$$\frac{1}{2}\left(\frac{mV_o}{M+m}\right)^2 = gH$$ ← solve for V_o

$$V_o^2 = (2gH)\left(\frac{M+m}{m}\right)^2$$

$$\boxed{V_o = \left(\frac{M+m}{m}\right)\sqrt{2gH} = \left(1+\frac{M}{m}\right)\sqrt{2gH}}$$

(b.) ellastic collision in one dim'l motion applies here

$V_m - V_M = V_{relative}$ $\qquad V_1$ = velocity of M after collision

$V_o - 0 = -(V_2 - V_1)$ $\qquad V_2$ = velocity of m after collision

Momentum conservation,

$mV_o = mV_2 + MV_1$ where $V_o = V_1 - V_2$

eliminate V_2 since it matters not, $V_2 = V_1 - V_o$

$mV_o = m(V_1 - V_o) + MV_1 \Rightarrow 2mV_o = (m+M)V_1$

P71 continued,

$$V_1 = \frac{2m V_0}{m + M}$$

$$\frac{1}{2} M V_1^2 = M g H$$ for upswing of M to $y = H$.

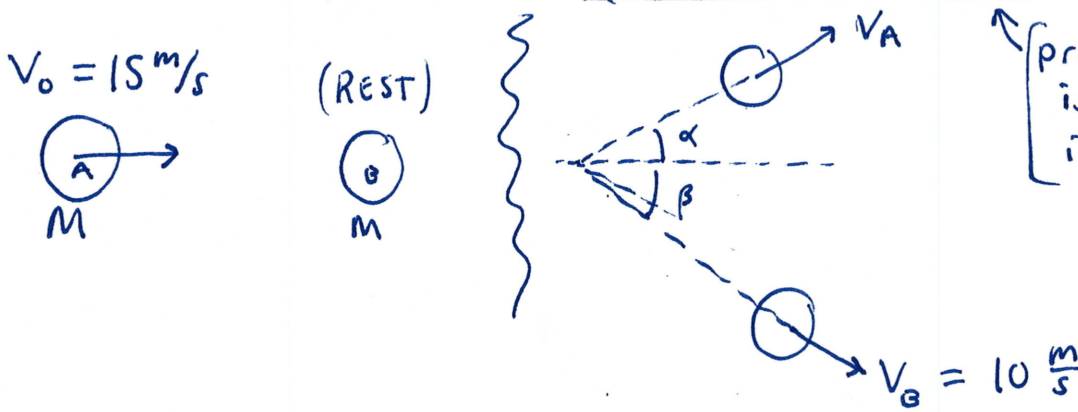
$$V_1^2 = 2 g H$$

$$\left(\frac{2m V_0}{m + M}\right)^2 = 2 g H$$

$$V_0^2 = \left(\frac{m+M}{2m}\right)^2 (2gH)$$

$$\boxed{V_0 = \left(\frac{m+M}{2m}\right)\sqrt{2gH} = \left(1 + \frac{M}{m}\right)\sqrt{\frac{gH}{2}}}$$

Problem 72: (2pts) Suppose an ice puck with velocity 15 m/s collides elastically with another identical puck which is at rest. The collision is off-center and thus the collision is not a head-on collision. If the puck initially at rest glides away with speed 10 m/s then what is the speed of the other puck and what angle is found between their paths after the collision?



[proved this is $\alpha+\beta = 90°$ in class, 3/6/23.]

$$M\langle V_o, 0\rangle = MV_A\langle \cos\alpha, \sin\alpha\rangle + MV_B\langle \cos\beta, -\sin\beta\rangle$$

I proved in class that $\alpha+\beta = 90°$ follows if we assume the collision is elastic. (that answers half of this question)

$$\langle 15, 0\rangle = \langle V_A\cos\alpha + 10\cos\beta,\ V_A\sin\alpha - 10\sin\beta\rangle$$

Elastic, $KE_{before} = KE_{after}$

$$\tfrac{1}{2}m(15\,m/s)^2 = \tfrac{1}{2}m(10\,\tfrac{m}{s})^2 + \tfrac{1}{2}m(V_A)^2$$

$$V_A = \sqrt{(225-100)}\ \tfrac{m}{s}$$

$$\boxed{V_A = 11.18\,m/s}$$

I'm curious, what are the values of α & β ?

$$15 = 11.18\cos\alpha + 10\cos\beta$$
$$0 = 11.18\sin\alpha - 10\sin\beta$$

But, $\beta = 90-\alpha$ and $\cos\beta = \cos(90-\alpha) = \sin(90)\sin\alpha = \sin\alpha$

and $\sin\beta = \sin(90-\alpha) = \sin(90)\cos(-\alpha) = \cos\alpha$

Thus $\underbrace{15 = 11.18\cos\alpha + 10\sin\alpha}_{\text{also solved by } \alpha = 41.81°}$ and $\underbrace{0 = 11.18\sin\alpha - 10\cos\alpha}$

$$\tan\alpha = \frac{10}{11.18}$$

$$\boxed{\alpha = 41.81°}$$

$\therefore$ $\boxed{\beta = 48.19°}$

give 5pts bonus to anyone who gets these