Your solutions should be neat, correct and complete. Same instructions as Mission 1 apply here.

Recommended Homework from Textbook: problems:

Chapter 38 #'s 3, 13, 23, 29, 31

Suggested Reading the following resources may be helpful:

(a.) Chapter 38 of the required text.

Problem 73: (2pts) What famous null result helped make many lose faith in the concept of aether ?

The Michelson Morley Experiment

Problem 74: (2pts) What are the two basic Axioms of Einstein's Theory of Special Relativity ?

1.) laws of physics are the same in any inertial frame of reference

2.) the speed of light in the vacuum is the same in every inertial frame of reference

Remark: the wording here could vary a lot.

Problem 75: (2pts) What is General Relativity and how is it different than Special Relativity ? What technology requires a General Relativitistic correction for accuracy ?

(1915) General Relativity is Einstein's theory of gravity

(1905) Special Relativity is Einstein's theory of modified classical mechanics which is consistent with E&M

GR is governed by $\underbrace{G_{\mu\nu}}_{\text{geometry}} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{energy/momentum}}$

GR $\Rightarrow$ redshift of light/radio etc. $\Rightarrow$ $\boxed{\text{GPS correction needed}}$

Problem 76: (2pts) Suppose a space train going $v = c/2$ with respect to frame S. This space train has a bike rider who rides on the train at $v_2 = c/2$ with respect to the frame of reference of the train. What is the observed velocity of the bike with respect to S ?

$$u = \frac{u' + V}{1 + \frac{u'V}{c^2}} = \frac{V_2 + V}{1 + \frac{V_2 V}{c^2}} = \frac{c/2 + c/2}{1 + \frac{c^2}{4c^2}}$$

$$u = \frac{c}{5/4}$$

$$u = \frac{4c}{5} \qquad c \cong 3 \times 10^8 \ m/s$$

$$\boxed{u = 0.8c} = \underline{2.4 \times 10^8 \ m/s}$$

Problem 77: (2pts) A given elementary particle is seen to last an average of 20 times as long as its usual lifetime as it is shot with high speed v from a particular high energy accelerator. What is the typical speed v of these particles ? (give answer in terms of speed of light c)

$$T_{LAB} = 20\, T_{REST\ FRAME} \qquad \underbrace{T = \gamma T'}_{\text{time dilation}}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 20 \qquad \text{where } \beta = V/c$$

$$\frac{1}{1-\beta^2} = 20^2 = 400$$

$$1 = 400 - 400\beta^2$$

$$400\beta^2 = 399$$

$$\beta^2 = \frac{399}{400} = \frac{V^2}{c^2}$$

$$\boxed{V = c\sqrt{\frac{399}{400}} \cong 0.9987c}$$

Problem 78: (2pts) A proton has $v = c/9$ find the relativistic kinetic energy of the proton and find its relativistic momentum.

$$m_p = 1.6726 \times 10^{-27}\ kg$$

$$KE = m_p c^2 (\gamma - 1) \qquad \gamma = \frac{1}{\sqrt{1-(c/9)^2/c^2}} = \frac{1}{\sqrt{1-\frac{1}{81}}}$$

$$= 1.6726 \times 10^{-27}$$

$$\gamma \cong 1.00623059$$

$$= (1.6726 \times 10^{-27}\ kg)(3 \times 10^8\ m/s)^2 (1.00623059)$$

$$= \boxed{1.515 \times 10^{-10}\ J}$$

$$P = m\gamma v = (1.6726 \times 10^{-27}\ kg)(1.00623059)\left(\tfrac{1}{9} \times 3 \times 10^8\ \tfrac{m}{s}\right)$$

$$= \boxed{5.610 \times 10^{-20}\ \tfrac{kg\,m}{s}}$$

Problem 79: (2pts) If a particle has relativitic kinetic energy which is 100 times its rest energy then how fast is the particle moving ?

$$KE = mc^2(\gamma - 1) = 100\,mc^2$$

$$\gamma - 1 = 100$$

$$\gamma = 101 = \frac{1}{\sqrt{1-\beta^2}}$$

$$(101)^2 = \frac{1}{1-\beta^2}$$

$$10201(1-\beta^2) = 1$$

$$10200 = 10201\,\beta^2$$

$$\beta^2 = \frac{10200}{10201} \Rightarrow V = c\sqrt{\frac{10200}{10201}}$$

$$\boxed{V \cong 0.99995098 4 c}$$

Problem 80: (2pts) Suppose event $E1$ has $t_1 = 10\,s$ and $x_1 = 10\,m$ and event $E2$ has $t_2 = 20\,s$ and $x_2 = 10\,m$. Consider a frame of reference $S' : (t', x')$ which moves in the usual way at a velocity of $v = c/99$. Find the spacetime coordinates of $E1$ and $E2$ with respect to the S'-observer. Of what is this an example ?

$$E1': \quad t_1' = \gamma(t_1 - vx_1/c^2)$$
$$x_1' = \gamma(x_1 - vt_1)$$

$$\Delta t' = t_2' - t_1' = 10.00051019s = \gamma \Delta t$$
(time dilation)

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 1/(99)^2}} = 1.000051019$$

$$t_1 - vx_1/c^2 = 10s - \frac{c(10m)}{c^2} = 10\left(1 - \frac{1}{3\times10^8}\right)s = 9.999999967s$$

$$\therefore \boxed{t_1' = 10.00051016s}$$

$$x_1 - vt_1 = 10m - \frac{c}{99}10s = 10m\left(1 - \frac{3\times10^8}{99}\right) = -30303020.3m$$

$$\boxed{x_1' = -30304566.33m = -3.030456633\times10^8 m}$$

Likewise, $\boxed{t_2' = 20.00102035s}$ & $\boxed{x_2' = -60609142.67m = -6.0609142\times10^8 m}$

Problem 81: (2pts) Suppose event $E1$ has $t_1 = 10\,s$ and $x_1 = 10\,m$ and event $E2$ has $t_2 = 10\,s$ and $x_2 = 20\,m$. Consider a frame of reference $S' : (t', x')$ which moves in the usual way at a velocity of $v = c/99$. Find the spacetime coordinates of $E1$ and $E2$ with respect to the S'-observer.

Again $\gamma = 1.000051019$.

$$t_1' = \gamma\left(t_1 - \frac{vx_1}{c^2}\right) = 10\gamma s\left(1 - \frac{1}{99\times3\times10^8}\right) = 10\gamma s = \underline{10.0005109s}$$

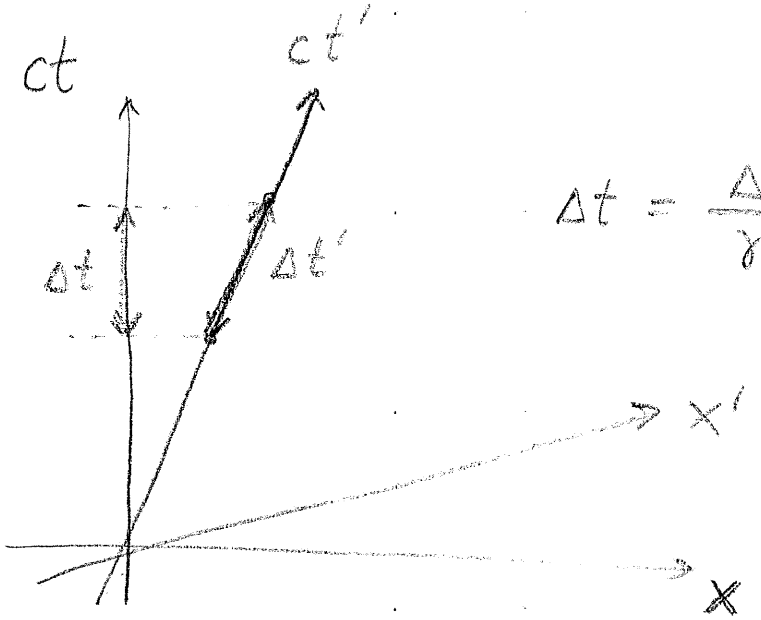
$$t_2' = \gamma\left(t_2 - \frac{vx_2}{c^2}\right) = \gamma\left(10s - \frac{20s}{99\times3\times10^8}\right) = 10\gamma s = \underline{10.0005109s}$$

$$x_1' = \gamma(x_1 - vt_1) = \gamma\left(10m - \frac{3\times10^8\times10m}{99}\right) = \underline{-30304566.3m}$$

$$x_2' = \gamma(x_2 - vt_2) = \gamma\left(20m - \frac{3\times10^8\times10m}{99}\right) = \underline{-30304556.3m}$$

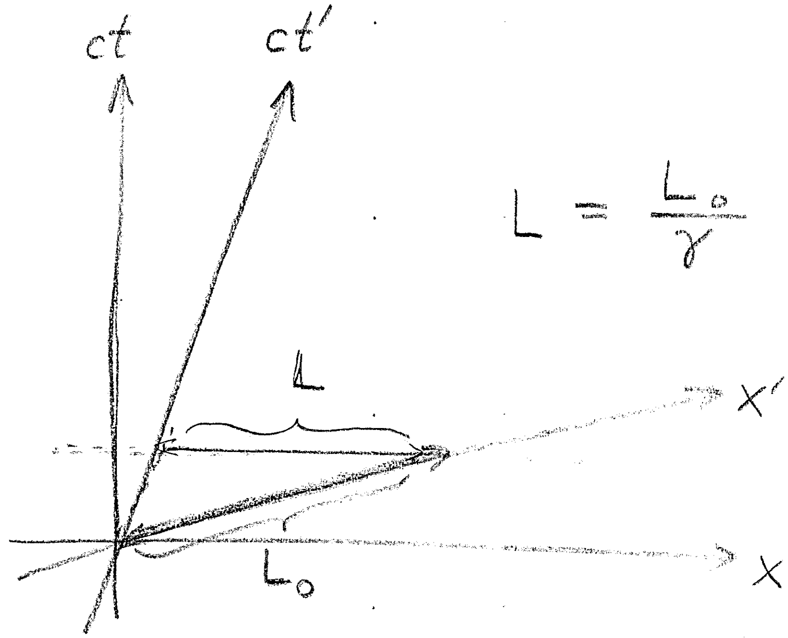
Remark: grader, please do not grade these too critically, my choice of $v = c/99$ makes γ so close to 1 that most calculators will be unable to capture details here. Moreover, $3\times10^8 m/s = c$ is an approximation, if students used $2.98\times10^8 m/s$ that'd change things a lot...

Problem 82: (2pts) Show time dilation with the appropriate spacetime diagram



$$\Delta t = \frac{\Delta t'}{\gamma}$$

Problem 83: (2pts) Show length contraction with the appropriate spacetime diagram



$$L = \frac{L_0}{\gamma}$$

Problem 84: (2pts) In the study of Spacetime in Special Relativity most physical equations are based on using the Minkowski metric. It is defined by:

$$g(\overline{v},\overline{w}) = -v_0w_0 + v_1w_1 + v_2w_2 + v_3w_3$$

where $\overline{v} = \langle v_0, v_1, v_2, v_3\rangle$ and $\overline{w} = \langle w_0, w_1, w_2, w_3\rangle$.

(a.) If $\overline{v} = \langle ct, ct, 0, 0\rangle$ then show $g(\overline{v},\overline{v}) = 0$. (left to reader ☺)

(b.) Let E_1, E_2 be events as observed in frame S and E_1', E_2' be the events observed in a frame S' which moves with velocity v in the x-direction in the S-frame. Show $g(E_2 - E_1, E_2 - E_1) = g(E_2' - E_1', E_2' - E_1')$ where $E_1 = (ct_1, x_1, y_1, z_1)$ etc.

$$\begin{aligned} ct_2' &= \gamma(ct_2 - \beta x_2) \quad \text{where } \beta = v/c \\ - \quad ct_1' &= \gamma(ct_1 - \beta x_1) \end{aligned}$$

$$\underline{c\Delta t' = \gamma(c\Delta t - \beta\Delta x)} \quad * \qquad \text{where } \begin{aligned} \Delta t' &= t_2' - t_1' \\ \Delta t &= t_2 - t_1 \\ \Delta x &= x_2 - x_1 \end{aligned}$$

Likewise,

$$\left.\begin{aligned} x_2' &= \gamma(x_2 - \beta t_2) \\ x_1' &= \gamma(x_1 - vt_1) \end{aligned}\right\} \Rightarrow \underline{\Delta x' = \gamma(\Delta x - v\Delta t)} \quad **$$

Notice $E_2' - E_1' = (c\Delta t', \Delta x', \Delta y', \Delta z')$ where $\Delta y' = y_2' - y_1'$ etc.

whereas $E_2 - E_1 = (c\Delta t, \Delta x, \Delta y, \Delta z)$ where $\Delta y = y_2 - y_1$ etc.

Thus, noting $y' = y$ and $z' = z$ for ☆,

$$\begin{aligned} g(E_2'-E_1', E_2'-E_1') &= -(c\Delta t')^2 + (\Delta x')^2 + \overbrace{(\Delta y')^2 + (\Delta z')^2}^{@} \\ &= -[\gamma(c\Delta t - \beta\Delta x)]^2 + [\gamma(\Delta x - v\Delta t)]^2 + @ \\ &= -\gamma^2(c^2\Delta t^2 - 2\beta c\Delta x\Delta t + \beta^2(\Delta x)^2) + \\ &\quad + \gamma^2((\Delta x)^2 - 2v\Delta x\Delta t + v^2(\Delta t)^2) + @ \\ &= (\Delta x)^2[1-\beta^2]\gamma^2 + (c\Delta t)^2[\beta^2 - 1]\gamma^2 + @ \\ &= -(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \qquad ☆ \\ &= g(E_2 - E_1, E_2 - E_1). \end{aligned}$$

We used $\gamma^2 = \dfrac{1}{1-\beta^2}$ thus $\gamma^2(1-\beta)^2 = \underline{1}$.