Your solutions should be neat, correct and complete. Same instructions as Mission 1 apply here.

Recommended Homework from Textbook: problems:

Chapter 10 #'s 1, 3, 5, 11, 13, 15, 17, 25, 37

Recommended Homework from Recommended Textbook (Young & Freedman, 9th ed): Chapter 9 (rotational motion) #'s 1, 11, 13, 15, 23, 27, 29, 31, 35, 37, 43, 57, 65, 73, 77, 79

Suggested Reading the following resources may be helpful:

(a.) Lectures 25, 26, 27 as posted on the course website,

(b.) Chapter 10 of the required text.

Problem 85: (2pts) Exercise 9-5 from Young and Freedman 9th edition.

$\theta(t) = \gamma t + \beta t^3$ where $\gamma = 0.800\ \text{rad}/s$ & $\beta = 0.0160\ \text{rad}/s^3$

(a.) $\omega = \frac{d\theta}{dt} = \boxed{\gamma + 3\beta t^2}$ ← angular velocity as function of t

(b.) $\omega_0 = \gamma = \boxed{0.800\ \text{rad}/s}$

(c.) $\omega(5) = \gamma + 3\beta(5s)^2 = \boxed{2\ \text{rad}/s}$

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta(5) - \theta(0)}{5} = \frac{0.8(5) + (0.016)(5)^3}{5}\frac{\text{rad}}{s} = \boxed{1.2\ \frac{\text{rad}}{s}}$$

$\omega(5) > \omega_{av}$ since $\alpha = \frac{d\omega}{dt} = 6\beta t$ this makes ω increase furtur for larger t.

Problem 86: (2pts) Exercise 9-9 from Young and Freedman 9th edition.

wheel turns with constant $\alpha = 0.450 \ rad/s^2$

(a.) $\omega_f = \omega_0 + \alpha t = \alpha t$ since wheel starts from rest $\omega_0 = 0$

To get to $\omega_f = 8.00 \ rad/s$

$$8.00 \ rad/s = (0.450 \ rad/s^2)\, t$$

$$\boxed{t = 17.78s}$$

(b.) $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 = \underline{71.13 \ rad}$

$$\Delta\theta = (71.13 rad)\left(\frac{1 rev.}{2\pi rad}\right) = \boxed{11.32 \text{ revolutions.}}$$

Problem 87: (2pts) Exercise 9-17 from Young and Freedman 9th edition.

At $t = 0$ have wheel with $\omega_0 = 24.0 \ rad/s$

Given constant $\alpha_1 = 60.0 \ rad/s^2$ until $t = 2.00 s$

Then it turns through 432 rad as it coasts to a <u>stop</u>

at constant α_2 (not given)

(a.) <u>total angle ?</u>

$$\Delta\theta_1 = \omega_0 (2.00s) + \frac{1}{2}(60.0 \ rad/s^2)(2.00s)^2 = 168 \ rad.$$

$$\text{Total } \Delta\theta = 168 rad + 432 rad = \boxed{600 rad}$$

(b.) <u>when did wheel stop ?</u> $\omega_1 = \omega_0 + (2.00s)(60 \ rad/s^2) = 144 \ rad/s$

$\omega_2 = 144 \ rad/s + \alpha_2 (t - 2)$ for $2 \le t$

$\omega_2^2 = (144 \ rad/s)^2 + 2\alpha_2 \Delta\theta_2 = 0 \rightarrow \alpha_2 = \frac{-(144 \ rad/s)^2}{2(432)} = -24 \frac{rad}{s^2}$

$0 = 144 \frac{rad}{s} - 24(t-2) \longrightarrow \boxed{t = 8s}$ (6 seconds of coasting)

$24t = 144 + 48$

$t = \frac{192}{24}$

(c.) $\boxed{\alpha_2 = -24 \frac{rad}{s^2}}$

Problem 88: (2pts) Exercise 9-25 from Young and Freedman 9th edition.

flywheel $R = 0.200\,m$, $\omega_0 = 0$, $\alpha = 0.600\ \frac{rad}{s^2}$

Compute magnitude of tangential & radial acceleration and the resultant acceleration of a point on its rim,

a.) at the start

b.) after turned through 120°

c.) after turned through 240°

$V = \omega R \rightarrow v^2 = \omega^2 R^2$

$\frac{V^2}{R} = \omega^2 R$

$$a = \sqrt{(V^2/R)^2 + a_T^2} = \sqrt{\omega^4 R^2 + R^2\alpha^2} = R\sqrt{\omega^4 + \alpha^2}$$

Here $\omega = \omega_0 + \alpha t = \alpha t$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta \rightarrow \omega = \sqrt{2\alpha\Delta\theta}$

$\omega^4 = 4\alpha^2(\Delta\theta)^2$

(a.) $a = \sqrt{0 + R^2\alpha^2} = \boxed{0.12\ m/s^2}$, $a_T = 0.12\ m/s^2$, $a_r = 0$

(b.) $a = \sqrt{4\alpha^2(\Delta\theta)^2 + \alpha^2}\,R = \left(\sqrt{4(\Delta\theta)^2 + 1}\right)\alpha R = \boxed{0.517\ m/s^2}$

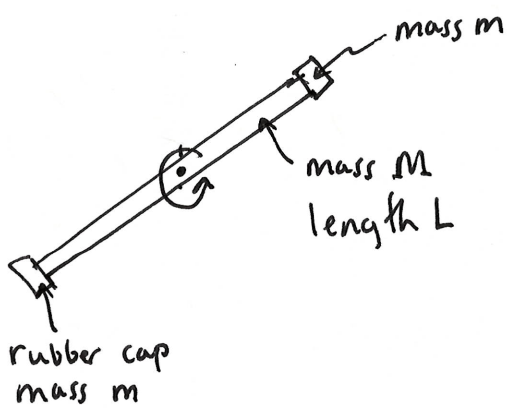
$a_T = 0.12\ m/s^2$

$a_r = 0.503\ m/s^2$

(c.) $a = \alpha R\sqrt{1 + 4(\Delta\theta)^2} = \boxed{1.01\ m/s^2}$ $a_T = 0.12\ m/s^2$

$a_r = \sqrt{a^2 - a_T^2} = 1.00\ m/s^2$

Problem 89: (2pts) Exercise 9-33 from Young and Freedman 9th edition.



$$I = \frac{1}{12}ML^2 + m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2$$

$$\boxed{I = \left(\frac{1}{12}M + \frac{1}{2}m\right)L^2}$$

Problem 90: (2pts) Exercise 9-59 from Young and Freedman 9th edition.

Printing press roller $\theta(t) = \gamma t^2 - \beta t^3$ where $\gamma = 3.20 \ \text{rad}/s^2$, $\beta = 0.400 \ \text{rad}/s^3$

(a.) $\omega = \frac{d\theta}{dt} = \boxed{2\gamma t - 3\beta t^2}$ angular velocity at time t.

(b.) $\alpha = \frac{d\omega}{dt} = \boxed{2\gamma - 6\beta t}$ angular acceleration at time t.

(c.) maximum positive ω? When does it happen?

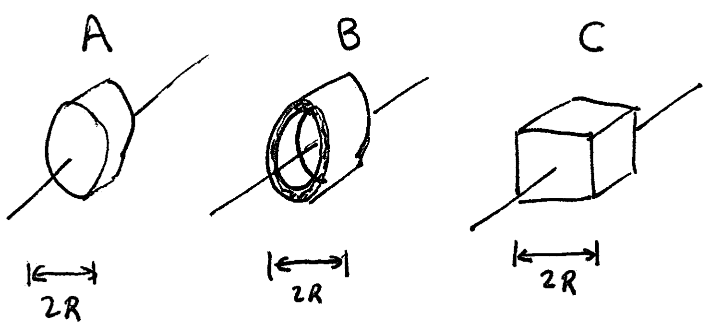
$$\frac{d\omega}{dt} = 0 = 2\gamma - 6\beta t \Rightarrow t = \frac{2\gamma}{6\beta} \cong \boxed{2.67 s}$$

$$\omega'' = -6\beta < 0 \therefore \omega(2.67s) = \max \omega.$$

$$\omega(2.67s) \cong 2\gamma(2.67) - 3\beta(2.67)^2$$

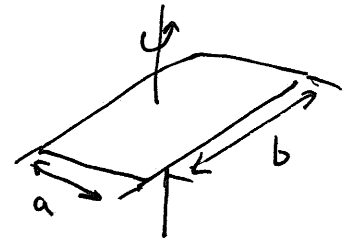
$$\boxed{\omega_{max} = 8.53 \ \text{rad}/s}$$

Problem 91: (2pts) Exercise 9-68 from Young and Freedman 9th edition.



equal masses

each mass m



(a.) A has smallest moment of inertia

(b.) B has largest moment of inertia

$$I = \frac{1}{12} M(a^2+b^2)$$

$$\downarrow a = b = 2R$$

$$I_c = \frac{1}{12} M((2R)^2 + (2R)^2)$$

$$I_c = \frac{8}{12} MR^2$$

(c.) $I_{sphere} = \frac{2}{5} mR^2$

$I_A = \frac{1}{2} mR^2$

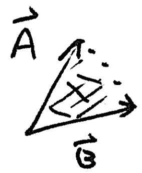
$I_B \approx mR^2$

$I_C = \frac{2}{3} mR^2$

$$\therefore \boxed{I_{sphere} < I_A < I_C < I_B}$$

Problem 92: (2pts) Let $\vec{A} = \langle 1, 3, 2\rangle$ and $\vec{B} = \langle -1, 0, 5\rangle$. Calculate $\vec{A} \times \vec{B}$. Also, find the area of the triangle which takes $\vec{A}$ and $\vec{B}$ as adjacent sides from a common vertex.

$$\vec{A} \times \vec{B} = \langle 1, 3, 2\rangle \times \langle -1, 0, 5\rangle$$
$$= \langle 3(5) - 2(0),\ 2(-1) - 1(5),\ 1(0) - 3(-1)\rangle$$
$$= \boxed{\langle 15, -7, 3\rangle}$$

area 

$$= \frac{1}{2}\,\|\vec{A} \times \vec{B}\|$$
$$= \frac{1}{2}\sqrt{15^2 + 7^2 + 3^2}$$
$$\cong \boxed{16.82}$$

Problem 93: (2pts) Suppose a force $\vec{F} = (3.0\,N)\hat{\mathrm{x}} - (2.0\,N)\hat{\mathrm{z}}$ is applied at the point $(2, 3, 8)m$. Find the torque produced by $\vec{F}$ about the origin.

$$\vec{\tau} = \vec{r} \times \vec{F} = \langle 2, 3, 8\rangle \times \langle 3, 0, -2\rangle\ \mathrm{Nm}$$
$$= \langle 3(-2) - 8(0),\ 8(3) - 2(-2),\ 2(0) - 3(3)\rangle\ \mathrm{Nm}$$
$$= \langle -6, 28, -9\rangle\ \mathrm{Nm}$$
$$= \boxed{\langle -6\,\mathrm{Nm},\ 28\,\mathrm{Nm},\ -9\,\mathrm{Nm}\rangle}$$

Problem 94: (2pts) Find the volume of the parallel-piped with sides $\vec{A} = \alpha\langle 1,1,2\rangle$ and $\vec{B} = \beta\langle 3,0,-3\rangle$ and $\vec{C} = \gamma\langle 2,3,5\rangle$ given $\alpha, \beta, \gamma > 0$. Is $\vec{A}, \vec{B}, \vec{C}$ a right-handed triple of vectors ?

$$\begin{aligned}
\text{Vol}(\vec{A},\vec{B},\vec{C}) &= \vec{A}\cdot(\vec{B}\times\vec{C}) \\
&= \vec{A}\cdot\left(\beta\gamma\langle 3,0,-3\rangle \times \langle 2,3,5\rangle\right) \\
&= \alpha\beta\gamma\langle 1,1,2\rangle \cdot \langle 0(5)+3(3), -3(2)-3(5), 3(3)-0(2)\rangle \\
&= \alpha\beta\gamma\langle 1,1,2\rangle \cdot \langle 9,-21,9\rangle \\
&= \alpha\beta\gamma(9-21+18) \\
&= \boxed{6\alpha\beta\gamma}
\end{aligned}$$

YES, $\vec{A}\cdot(\vec{B}\times\vec{C}) > 0 \Rightarrow \{\vec{A},\vec{B},\vec{C}\}$ is right-handed

Problem 95: (2pts) A force $\vec{\mathbf{F}} = (3.0\,N)(3\hat{\mathbf{x}} - 2\hat{\mathbf{z}})$ is applied to a point mass $M = 2.00\,kg$ which has initial velocity $\vec{\mathbf{v}} = (30\,m/s)(\hat{\mathbf{y}} + \hat{\mathbf{z}})$ at the point $(1,2,3)m$. Find the torque on M and the angular momentum of M with respect to the origin.

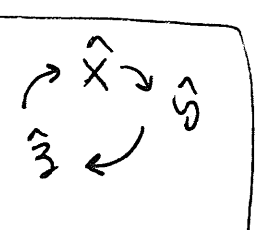
$$\vec{L} = m\vec{r}\times\vec{v} = (2.00kg)\langle 1,2,3\rangle \times \langle 0,1,1\rangle (30\tfrac{m}{s})m$$

$$\vec{L} = 60\frac{kgm^2}{s}\langle 2(1)-3(1), 3(0)-1(0), 1\cdot 1-2\cdot 0\rangle$$

$$\vec{L} = 60\frac{kgm^2}{s}\langle -1,-1,1\rangle$$

$$\boxed{\vec{L} = \langle -60,-60,60\rangle \frac{kgm^2}{s}}$$ angular momentum

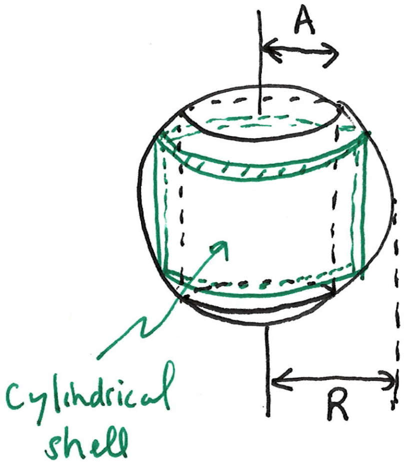
$$\vec{\tau} = \vec{r}\times\vec{F} = (\hat{x}+2\hat{y}+3\hat{z})\times(3\hat{x}-2\hat{z})(3.0N)m$$

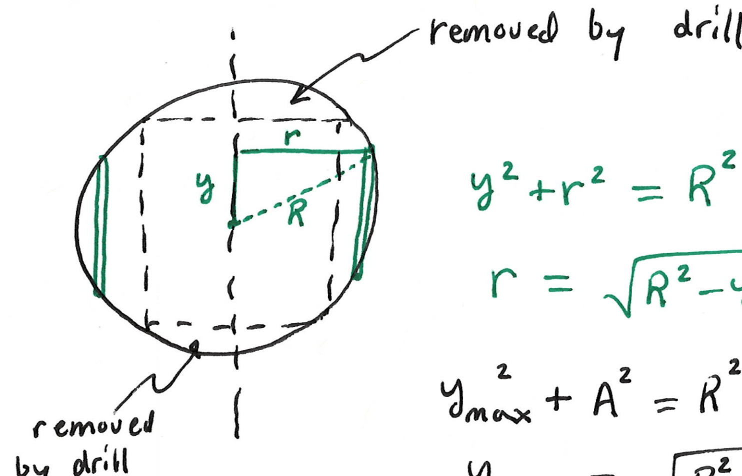


$$\begin{aligned}
&= (-2\hat{x}\times\hat{z} + 6\hat{y}\times\hat{x} - 4\hat{y}\times\hat{z} + 9\hat{z}\times\hat{x})\,3\,Nm \\
&= (2\hat{y} - 6\hat{z} - 4\hat{x} + 9\hat{y})\,3Nm \\
&= \boxed{\langle -4, 11, -6\rangle Nm}
\end{aligned}$$

(similar to E7 of Lecture 26)(included ↴)

Problem 96: (2pts) Calculate the moment of inertia for a napkin ring by the method of cylindrical shells. Assume the ring is manufactured from a sphere of wood with mass M and radius R drilled out with a cylindrical hole of radius A centered on a diameter of the sphere. Also, assume the axis of rotation is the diameter of the drill hole.





$$y^2 + r^2 = R^2$$
$$r = \sqrt{R^2 - y^2}$$
$$y_{max}^2 + A^2 = R^2$$
$$y_{max} = \sqrt{R^2 - A^2}$$
$$y = \sqrt{R^2 - r^2}$$
$$A \leq r \leq R$$

$$dV = (2\pi r)(2y)\,dr$$
$$dI = r^2\,dm = r^2 \rho\, dV$$

$$I = \int_A^R 4\pi\rho r^3 \sqrt{R^2 - r^2}\,dr$$

$$\boxed{u = R^2 - r^2, \; du = -2r\,dr \quad r^2 = R^2 - u, \; u(A) = R^2 - A^2, \; u(R) = 0}$$

$$= \frac{4\pi\rho}{-2}\int_{R^2-A^2}^{0} (R^2 - u)\sqrt{u}\,du$$

flipped bounds to absorb the minus

$$= 2\pi\rho \int_0^{R^2-A^2} \left(R^2\sqrt{u} - u^{3/2}\right)du$$
$$= 2\pi\rho\left(R^2 \cdot \frac{2}{3}(R^2-A^2)^{3/2} - \frac{2}{5}(R^2-A^2)^{5/2}\right)$$
$$= 2\pi\rho\,(R^2-A^2)^{3/2}\left[\frac{2}{3}R^2 - \frac{2}{5}(R^2-A^2)\right]$$
$$= 2\pi\rho\,(R^2-A^2)^{3/2}\left[\frac{4}{15}R^2 + \frac{2}{5}A^2\right]$$

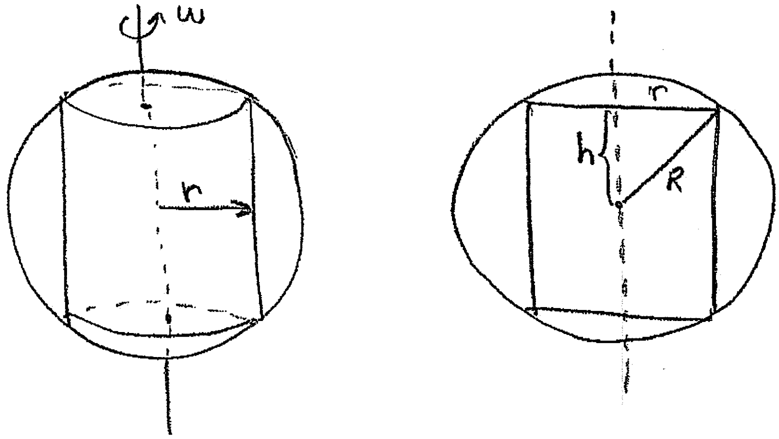
M is mass before drilling thus ↴

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

$$= \frac{(2\pi)(3M)}{4\pi R^3}(R^2-A^2)^{3/2}\left[\frac{4}{15}R^2 + \frac{2}{5}A^2\right]$$
$$= \frac{3}{2}M\,(R^2-A^2)^{3/2}\left[\frac{4}{15}\frac{R^2}{R^3} + \frac{2}{5}\frac{A^2}{R^3}\right]$$
$$= \boxed{M\,(R^2-A^2)^{3/2}\left[\frac{2}{5}\frac{1}{R} + \frac{3}{5}\frac{A^2}{R^3}\right]} \longrightarrow \frac{2}{5}MR^2 \text{ when } A = 0.$$

Remark: the answer may look different for students and still be correct!

$\boxed{E7}$ Find I for sphere of radius R rotated about a diameter



$$h = \sqrt{R^2 - r^2}$$

$$dV = \underbrace{(2\pi r)(2h)}_{\text{area of shell}}\underbrace{dr}_{\text{thickness}} \rightsquigarrow dm = 4\rho\pi r\sqrt{R^2 - r^2}\, dr$$

all at radius r from rotation axis, $dI = r^2\, dm$.

$$I = \int_0^R 4\rho\pi r^3 \sqrt{R^2 - r^2}\, dr$$

: Let $u = R^2 - r^2$
then $du = -2r\,dr \to r\,dr = \frac{-du}{2}$
and $u(0) = R^2$, $u(R) = 0$
note $r^2 = R^2 - u$

$$= \int_{R^2}^{0} 4\rho\pi r^2 \sqrt{u}\left(\frac{-du}{2}\right)$$

$$= \int_0^{R^2} 2\rho\pi\,(R^2 - u)\sqrt{u}\, du$$

$$= 2\left[\frac{M}{\frac{4}{3}\cancel{\pi} R^3}\right]\cancel{\pi}\left(\frac{2}{3}R^2 u^{3/2} - \frac{2}{5}u^{5/2}\right)\Bigg|_0^{R^2}$$

$$= \frac{3M}{2R^3}\left(\frac{2}{3}R^5 - \frac{2}{5}R^5\right)$$

$$\frac{2}{3} - \frac{2}{5} = \frac{10}{15} - \frac{6}{15} = \frac{4}{15}$$

$$\frac{4}{15}\cdot\frac{3}{2} = \frac{2}{5}$$

$$= \boxed{\frac{2}{5}MR^2 = I_{sphere}}$$

Well folks, I've done enough, see Table 9.1 for more. I will provide 9.1 for test 3, but I may ask you to derive one of those (as I have just done in $\boxed{E1}$ through $\boxed{E7}$).