Your solutions should be neat, correct and complete. Same instructions as Mission 1 apply here.

Recommended Homework from Textbook: problems:

Chapter 11 #'s 1, 3, 5, 9, 13, 27

Recommended Homework from Recommended Textbook (Young & Freedman, 9th ed): Chapter 10 (rotational dynamics) #'s 1, 5, 13, 15, 21, 25, 29, 33, 37, 39, 49, 53, 57, 61

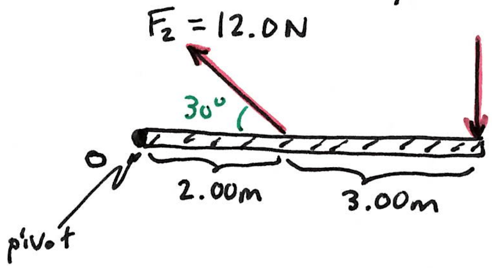
Suggested Reading the following resources may be helpful:

(a.) Lectures 28, 29, 30 as posted on the course website,

(b.) Chapter 11 of the required text.

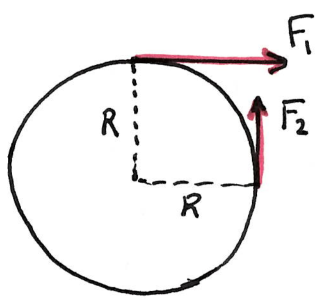
Problem 97: (2pts) Exercise 10-2 from Young and Freedman 9th edition.

Calculate net-torque about point O as pictured in Fig 10-35



$$\tau_{net} = +R_2 F_2 \sin(30) - R_1 F_1$$
$$= (2.00m)(12.0N)(0.5) - (5.00m)(8.00N)$$
$$= \boxed{-28.0 \text{ Nm}}$$ or (28 J in the CW-direction)

Problem 98: (2pts) Exercise 10-4 from Young and Freedman 9th edition.



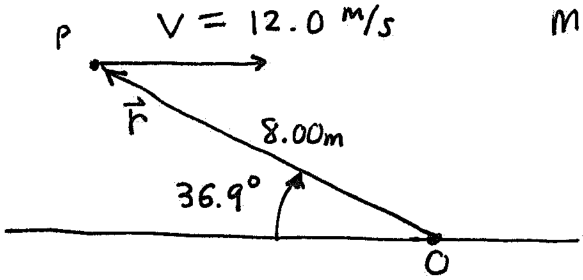
$$\tau_{net} = -RF_1 + RF_2$$
$$= (-0.33m)(8.60N) + (0.33m)(4.30N)$$
$$= \boxed{-1.419 \text{ Nm}}$$ (aka 1.419 J in CW-direction)

$R = 0.330m$

$F_1 = 8.60N$

$F_2 = 4.30N$

Problem 99: (2pts) Exercise 10-31 from Young and Freedman 9th edition.



$M = 0.300\ kg$

$\vec{r} = \langle -8\cos 36.9^\circ,\ 8\sin 36.9^\circ,\ 0 \rangle\ m$

$\vec{V} = \langle 12.0\ m/s,\ 0,\ 0 \rangle$

$$\vec{L} = m\vec{r} \times \vec{V} = (0.300\,kg)\left\langle 0,\ 0,\ -(8\sin 36.9^\circ\,m)(12.0\,m/s)\right\rangle$$

$$= \underbrace{\left\langle 0, 0, -17.29\ \tfrac{kg\,m^2}{s}\right\rangle}_{\text{actual angular momentum}} \longrightarrow \boxed{L = 17.29\ \tfrac{kg\,m^2}{s}}$$

(magnitude) of angular momentum.

Problem 100: (2pts) Exercise 10-47 from Young and Freedman 9th edition.

Net torque of 20.0 Nm exterted on wheel for a total of 8.00s during which time ω goes from zero to 100 rev/min. Then the external torque is removed and the friction in its bearing slows the wheel to a stop in 70.0 s

(a.) find moment of inertia
(b.) frictional torque
(c.) # of revolutions made in last 70 seconds.

(a.) $$\alpha_1 = \frac{\omega_f - \omega_0}{\Delta t_1} = \left(\frac{100\ rev}{min}\right)\left(\frac{1}{8.00s}\right)\left(\frac{min}{60s}\right)\left(\frac{2\pi\ rad}{1\ rev}\right) = 1.309\ \frac{rad}{s^2}$$

$$\tau = I\alpha_1 \quad \therefore \quad I = \frac{\tau}{\alpha_1} = \frac{20.0\,Nm}{1.309\ rad/s^2} = \boxed{15.28\ kg\,m^2}$$

(b.) $$\alpha_2 = \frac{0 - \omega_f}{\Delta t_2} = -\left(\frac{100\ rev}{min}\right)\left(\frac{1}{70.0s}\right)\left(\frac{min}{60s}\right)\left(\frac{2\pi\ rad}{1\ rev}\right) = -0.1496\ \frac{rad}{s^2}$$

$$\tau_{friction} = I\alpha_2 = (15.28\ kg\,m^2)\left(-0.1496\ \frac{rad}{s^2}\right) = -2.286\ Nm$$

$$\boxed{|\tau_{friction}| = 2.286\ Nm}$$

(c.)

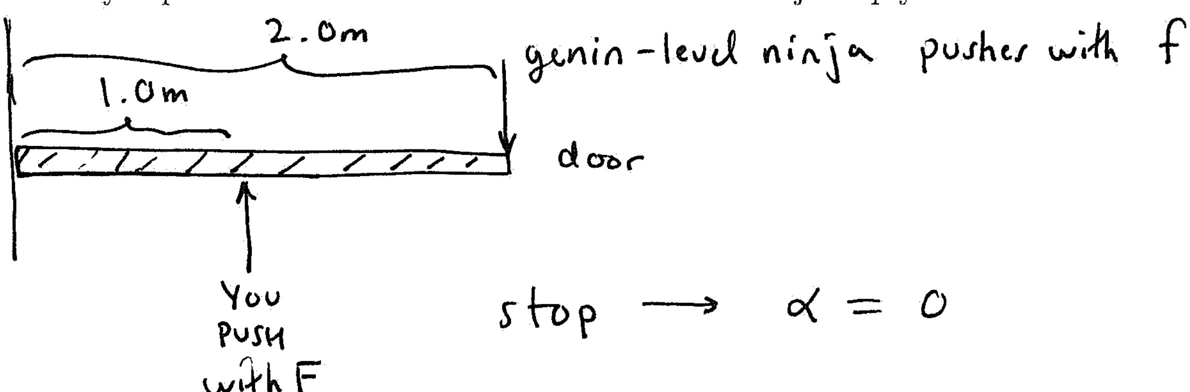
$$\omega_{end}^2 = \omega_f^2 + 2\alpha_2\Delta\theta \longrightarrow \Delta\theta = \frac{-\omega_f^2}{2\alpha_2} = \frac{-\left(100\ \frac{rev}{min}\cdot\frac{min}{60}\cdot\frac{2\pi\ rad}{rev}\right)^2}{2\left(-0.1496\ \frac{rad}{s^2}\right)}$$

$$\Delta\theta = (366.52\ rad)\left(\frac{rev}{2\pi\ rad}\right) = \boxed{58.33\ \text{revolutions}}$$

Problem 101: (2pts) Exercise 10-59 from Young and Freedman 9th edition.

(OMIT) (ALSO, solved in Lecture on 4/19/23)

Problem 102: (2pts) You push the edge of a door of large square door with side-length 2.00 m at the middle of the door. A mischevious genin-level ninja who just learned about mechanical advantage pushes at the edge of the door and stops your push with a smaller force. If you push with force F then what force did the ninja stop you ?



stop $\longrightarrow$ $\alpha = 0$

$$\tau_{net} = I\alpha = 0$$

Net torque is $\underline{\text{zero}}$,

$$\tau_{net} = (1.0\,m)F - (2.0\,m)f$$

$$f = \frac{(1.0\,m)F}{2.0\,m} = \frac{F}{2}$$

$$\boxed{f = F/2}$$

Problem 103: (2pts) A wheel is given $\alpha = 2.3\ rad/s^2$. If the wheel is initially at rest then find:

(a.) the angular velocity after $2.0\ s$

(b.) the angle through which the wheel turns in the first two seconds

(c.) the torque on the wheel given that $I = 2.0\ kg\,m^2$.

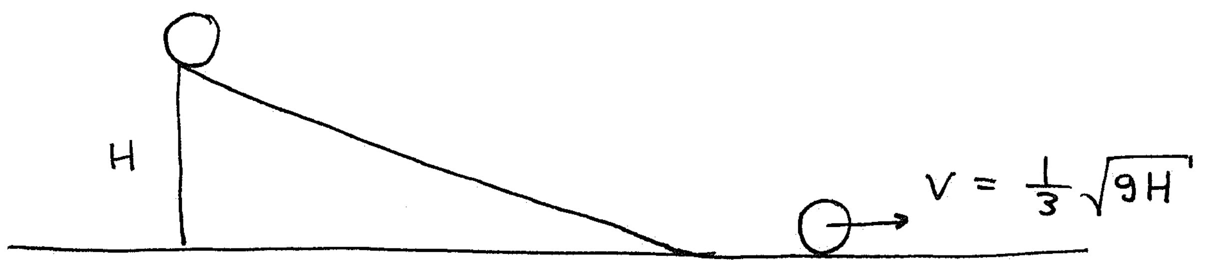
(a.)
$$\omega_f = \omega_0 + \alpha t$$
$$= 0 + \left(2.3\ \tfrac{rad}{s^2}\right)(2.0s)$$
$$= \boxed{4.6\ rad/s}$$

(b.)
$$\Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$= 0(2s) + \tfrac{1}{2}(2.3\ rad/s^2)(2.0s)^2$$
$$= \boxed{4.6\ \text{radians}}$$

(c.)
$$\tau = I\alpha$$
$$= (2.0\ kg\,m^2)\left(2.3\ \tfrac{rad}{s^2}\right)$$
$$= \boxed{4.6\ Nm}$$

Problem 104: (2pts) A mass rolls down an inclined plane without slipping a vertical distance H. If the translational speed of the mass is $\frac{1}{3}\sqrt{gH}$ then what is the moment of inertia for the mass?

(initially at rest)



$$MgH = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2 \quad , \quad V = \omega R$$

$$2MgH = M\omega^2 R^2 + I\omega^2$$

$$I = \frac{2MgH - M\omega^2 R^2}{\omega^2}$$

$$= \frac{(2M)(9V^2) - M\omega^2 R^2}{\omega^2}$$

$$= \frac{18M\omega^2 R^2 - M\omega^2 R^2}{\omega^2}$$

$$= \boxed{17MR^2}$$

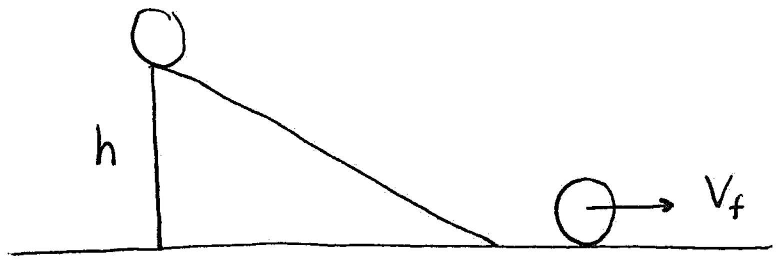
$V = \frac{1}{3}\sqrt{gH}$

$3V = \sqrt{gH}$

$9V^2 = gH$

$V = \omega R$

Problem 105: (2pts) Find the speed at which a solid sphere rolls after it rolls down an inclined plane of height h without slipping given that it is initially at rest. If the mass of the sphere is M and its radius is R find its final speed v as reaches the base of the incline.



$$Mgh = \frac{1}{2}MV_f^2 + \frac{1}{2}I\omega_f^2$$

$$Mgh = \frac{1}{2}MV_f^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V_f}{R}\right)^2$$

$$gh = \left(\frac{1}{2} + \frac{1}{5}\right)V_f^2 = \frac{7}{10}V_f^2$$

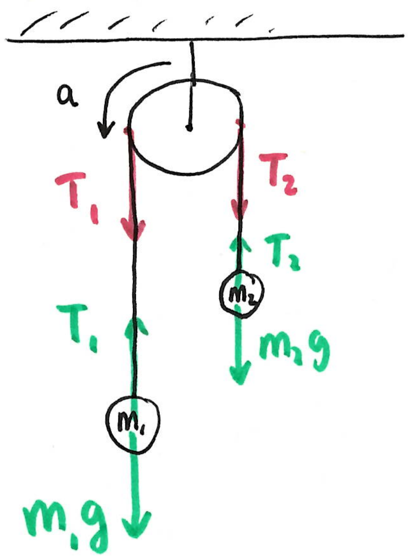
$$V_f^2 = \frac{10gh}{7}$$

$$\boxed{V_f = \sqrt{\frac{10gh}{7}}}$$

$\Delta y = \frac{1}{2} a \Delta t^2 \qquad a = \frac{2(4.0m)}{(2.0s)^2} = 2\,m/s^2$

Problem 106: (2pts) A mass of $m_1 = 100\,kg$ and another of mass $m_2 = 50\,kg$ are attached to a light cable which wraps around a pulley without slipping. Let I be the moment of inertia of the pulley. Starting from rest you observe m_1 fall $4.0\,m$ in a time of $2.0\,s$. What is the value of I ?

(gave $R = 20cm = 0.2m$ in Canvas)



$a = R\alpha$

$\alpha = \frac{a}{R}$

$$I\alpha = RT_1 - RT_2 \rightarrow \frac{I}{R^2} a = T_1 - T_2$$

$$+\begin{pmatrix} m_1 a = m_1 g - T_1 \\ m_2 a = T_2 - m_2 g \end{pmatrix}$$

$$(m_1 + m_2)a = (m_1 - m_2)g + T_2 - T_1$$

$$(m_1 + m_2)a = (m_1 - m_2)g - \frac{I}{R^2} a$$

$$\frac{Ia}{R^2} = (m_1 - m_2)g - (m_1 + m_2)a$$

$$I = \left[\frac{(m_1 - m_2)g - (m_1 + m_2)a}{a}\right] R^2$$

$$= \left[\frac{(50kg)(9.8\,m/s^2) - (150)kg(2\,m/s^2)}{2.0\,m/s^2}\right](0.2m)^2 = \boxed{3.8 kgm^2}$$

Problem 107: (2pts) A yo-yo has $300\,J$ of energy in the form of rotational kinetic energy. The yo-yo also has an angular momentum of $L = 20\,m^2 kg/s$. What is the moment of intertia of the yo-yo ?

$$KE = \frac{1}{2} I w^2 \qquad L = Iw$$

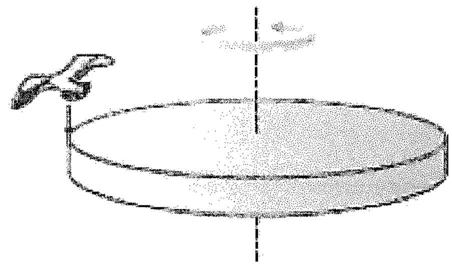
$$w = L/I$$

$$KE = \frac{1}{2} I \left(\frac{L}{I}\right)^2 = \frac{L^2}{2I}$$

$$I = \frac{L^2}{2KE} = \frac{(20\,m^2 kg/s)^2}{2(300\,J)} \cong \boxed{0.6666\,kgm^2}$$

$$\frac{2}{3} kgm^2$$

Problem 108: (2pts) Consider a cylindrical turntable whose mass is M and radius is R, turning with an initial angular speed ω_1.



$M_p = M_b$

sorry.

(a.) A parakeet of mass M_p, after hovering in flight above the outer edge of the turntable, gently lands on it and stays in one place on it, as shown above. What is the angular speed of the turntable after the parakeet lands? (Use M,M_b and ω_1 in your answer.)

(b.) Becoming dizzy, the parakeet jumps off (not flies off) with a velocity relative to the turntable. The direction of is tangent to the edge of the turntable and in the direction of its rotation. What will be the angular speed of the turntable afterwards? Express your answer in terms of the two masses M_b and M, the radius R, the parakeet speed $v_jump = v_J$ and the initial angular speed ω_1.

(a.)

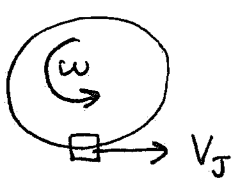
$$L_o = L_f$$

$$I_o \omega_o = I_f \omega_f$$

$$\frac{1}{2} M R^2 \omega_1 = \left(\frac{1}{2} M R^2 + M_p R^2\right) \omega_f$$

$$\omega_f = \left(\frac{\frac{1}{2} M R^2}{\frac{1}{2} M R^2 + M_p R^2}\right) \omega_1 = \boxed{\left(\frac{M}{M + 2M_p}\right) \omega_1}$$

(b.)



$$L_o = L_2 = L_{disk} + L_{bird}$$

$$I_o \omega_1 = \left(\frac{1}{2} M R^2\right) \omega_2 + \left(M_p R^2\right) \omega_2$$

$$\frac{1}{2} M R^2 \omega_1 = \frac{1}{2} M R^2 \omega_2 + M_p R^2 \omega_2$$

$$M \omega_1 = M \omega_2 + 2 M_p \omega_2 = (M + 2M_p) \omega_2$$

$$\boxed{\omega_2 = \frac{M \omega_1}{M + 2M_p}}$$