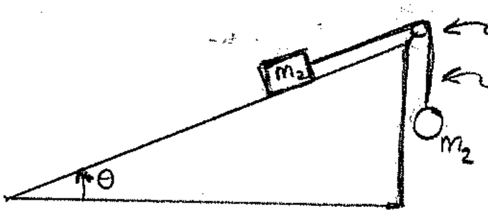
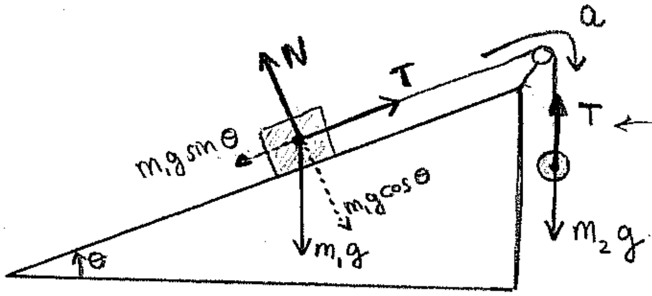
LECTURE 10

- HERE WE CONTINUE TO STUDY NEWTON'S 2^{nd} LAW AND ITS MANY APPLICATIONS.

E1 Suppose a mass m_1 pulls a mass m_2 as pictured below. Find the acceleration of m_2 in the $\parallel$-direction.



Draw free-body diagrams: (we should apply Newton's 2^{nd} Law to both masses to solve this problem)



m_2: $\quad m_2 a = m_2 g - T$

m_1: $\quad m_1 a = T - m_1 g \sin\theta$ (parallel to plane)

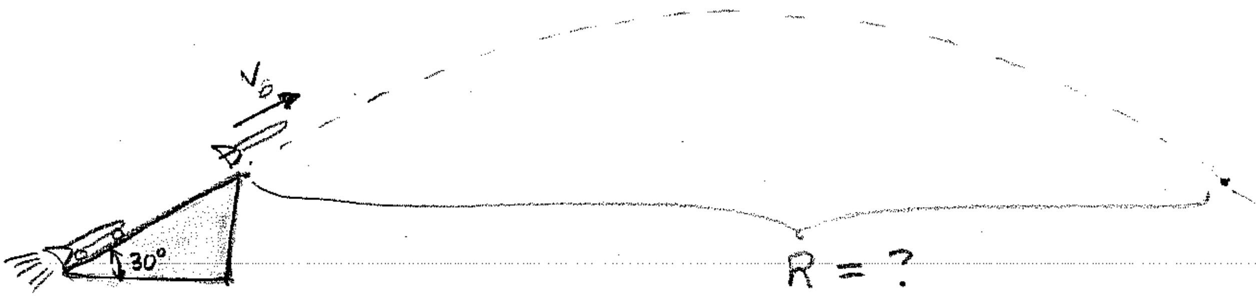
Add equations to find

$$m_1 a + m_2 a = m_2 g - m_1 g \sin\theta$$

$$\therefore \quad \boxed{a = g\left(\frac{m_2 - m_1 \sin\theta}{m_1 + m_2}\right)}$$

If $a > 0$ then the box slides up the plane, if $a < 0$ then the box slides down. Notice we need only consider the sign of $m_2 - m_1 \sin\theta$. For example, if $\theta = 90°$ then $a = 0$ if $m_1 = m_2$, or $a > 0$ if $m_2 > m_1$, or $a < 0$ if $m_2 < m_1$. Does this seem reasonable?

E2 Suppose a rocket outputs a constant thrust which produces an force equal to ten times it weight on a level track. If the rocket fires for 1.00s seconds as it travels a ramp at $\theta = 30°$ above horizontal and then it turns off and flies as a projectile then how far does it fly horizontally from the end of the ramp?



For the first second of motion: $\vec{F}_{net} = \vec{F}_{rocket} + \vec{F}_g$

only the parallel component goes into making V_0.

$$\left(\vec{F}_{net}\right)_{\parallel} = F_{rocket} - mg\sin 30°$$

$$= 10mg - mg/2$$

$$= 19mg/2 \quad \Rightarrow \quad a = 19g/2 \quad \text{(parallel to plane)}$$

To find V_0 we note $V_0 = at = \frac{19}{2}(9.81\,\frac{m}{s^2})(1.00s) = \underline{93.2\,\frac{m}{s}}$.

We found $R(\theta) = \dfrac{V_0^2 \sin(2\theta)}{g}$ in a previous lecture,

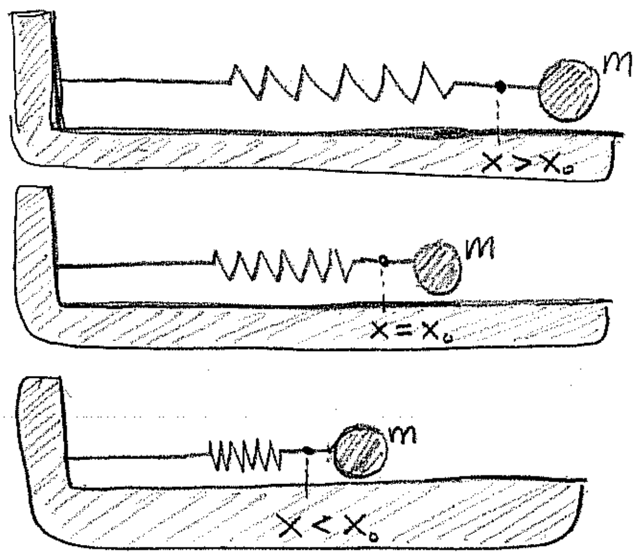
$$R = \frac{(93.2\,\frac{m}{s})^2 \sin(60)}{9.81\,m/s^2} = \boxed{766.7\,m}$$

SPRING FORCE

Let x be the position of a spring as measured from its equilibrium $(x = x_0)$ then Hooke's Law states that the spring exterts

$$\boxed{F = -k(x - x_0)}$$

k = stiffness



stretched:

$F = -k(x - x_0) < 0$

spring pulls left on m

Equilibrium:

$F = -k(x_0 - x_0) = 0$

Compressed:

$F = -k(x - x_0) > 0$

spring pushes right on mass m.

If we imagine the spring push/pulling on a mass m which slides along a frictionless surface then we have that

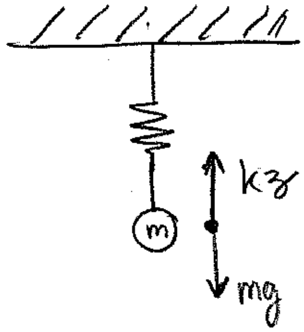
$$ma = -k(x - x_0)$$

If we let $x_0 = 0$ then $a = \frac{d^2x}{dt^2}$ and

$$m\frac{d^2x}{dt^2} = -kx$$

We'll discuss this later since the mathematics requires some discussion. For now, with springs we mainly consider <u>static</u> cases.

E3 Suppose a spring has $k = 10\ \mathrm{N/cm}$. If the spring holds up a mass $m = 300g$ then how far does the spring stretch? (assume static)

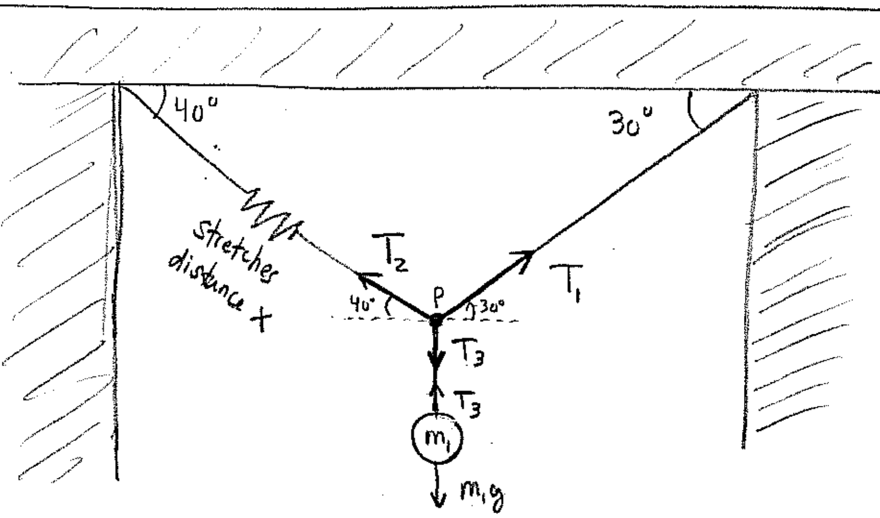


after spring stretched the configuration is assumed static so we know $a = 0$ the spring force and gravity balance hence $kz = mg$

$$z = \frac{mg}{k} = \frac{(0.3kg)(9.81\frac{m}{s^2})}{(10\frac{N}{cm})(\frac{100cm}{1m})}$$

$$\therefore\ z = 2.943 \times 10^{-3}\ m \quad \text{or} \quad \boxed{z = 2.94\ mm}$$

E4 Suppose $m_1 = 3kg$ is suspended motionless as pictured. If the spring has $k = 10\ \mathrm{N/cm}$ then how far does it stretch?



Since the system is in static equilbrium $\vec{F}_{net} = m\vec{a} = \vec{0}$, which means the forces must balance.

m_1: $m_1 g = T_3$

P : $T_3 = T_{1y} + T_{2y}$ & $T_{2x} = T_{1x}$ → $m_1 g = T_1 \sin 30 + T_2 \sin 40$; $T_1 \cos 30 = T_2 \cos 40$

Note $T_1 = T_2\left(\frac{\cos 40}{\cos 30}\right)$ and so $m_1 g = T_2\left(\frac{\cos 40}{\cos 30}\right) + T_2 \sin 40$

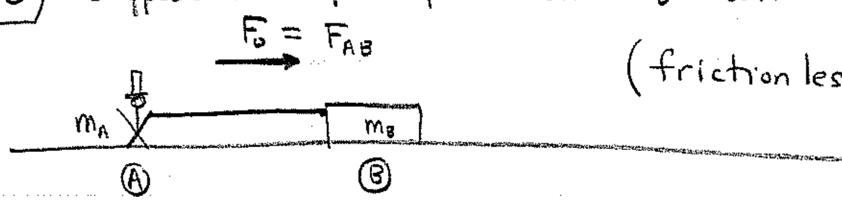
$$\therefore\ T_2 = \frac{m_1 g}{\sin 40 + \frac{\cos 40}{\cos 30}} = \frac{29.43N}{1.527} = 19.27N \quad \therefore\ x = \frac{T_2}{k} = \frac{19.27N}{(10\frac{N}{cm})(\frac{100cm}{m})}$$

$$\boxed{x = 0.0193m}$$

Newton's Third Law:

When bodies interact forces come in pairs. Moreover if $\vec{F}_{BA}$ is the force of B on A and $\vec{F}_{AB}$ is the force that A places on B then $\vec{F}_{BA} = -\vec{F}_{AB}$

E5 Suppose Mr. Tophat pushes on m_B with force $F_0 = F_{AB}$ (frictionless)



then m_B counteracts with $F_{BA} = -F_0$ right back (to the left) at the point of contact. We can ponder Newton's 2^{nd} Law in this context,

$$m_A \vec{a}_A = -m_A g\,\hat{j} + \vec{N}_A - F_0\,\hat{i}$$

$$m_B \vec{a}_B = -m_B g\,\hat{j} + \vec{N}_B + F_0\,\hat{i}$$

Horizontally we find,

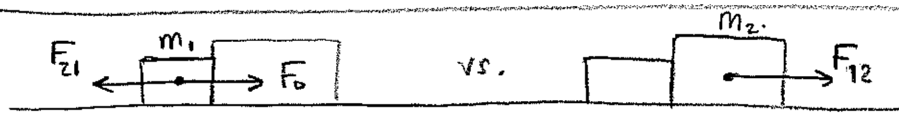
$$m_A a_{Ax} = -F_0 \longrightarrow \underline{a_{Ax} = -F_0/m_A}.$$

$$m_B a_{Bx} = F_0 \longrightarrow \underline{a_{Bx} = F_0/m_B}.$$

does this make sense to you?

E6 Push on m_1 & m_2 with force F_0. If we push on

then what is the contact force of m_1 on m_2?



by 3^{rd} Law! $F_{21} = -F_{12}$

$$m_1 a = F_0 + F_{21} \qquad m_2 a = F_{12}$$

Note $a = \dfrac{F_0 + F_{21}}{m_1} = \dfrac{F_{12}}{m_2}$ thus we find $m_2 F_0 + m_2 F_{21} = m_1 F_{12}$

(F_{12} is the contact force)

$$\therefore \boxed{F_{12} = \frac{m_2 F_0}{m_1 + m_2}}$$