LECTURE 11

- in this lecture we continue to explore Newton's 2nd Law and we add frictional forces to the discussion. Friction forces come in two varieties in this lecture:

I.) $F_{f,\,static} \leq \mu_s N$ (magnitude only!)

- the direction of the static friction force is such that static equilbrium is maintained
- notice the $\leq$, this indicates the static friction force can range from zero up to the max-value $\mu_s N$.
- N is magnitude of normal force and μ_s = coefficient of static friction.

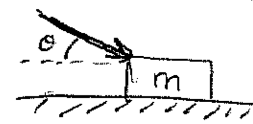
II.) $F_{f,\,kinetic} = \mu_k N$ (magnitude only!)

- the direction is opposite the motion. We can write $\vec{F} = -\mu_k N \hat{v}$ where $\hat{v} = \frac{1}{|\vec{v}|}\vec{v}$ is the unit-vector pointing in direction of velocity.
- $F_{f,\,kinetic} = \mu_k N$ is typically <u>smaller</u> than the maximal static friction force $\mu_s N$ for a given <u>interface</u>

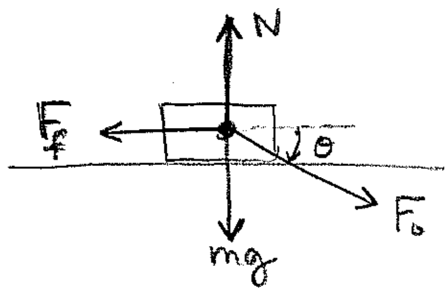
For both I & II the coefficients μ_s, μ_k depend on many factors. For a given pair of surfaces we're usually given μ_s, μ_k. See page 130 for a table of coefficients, for example:

steel on steel :	$\mu_s = 0.7$,	$\mu_k = 0.6$
teflon on teflon :	$\mu_s = 0.04$,	$\mu_k = 0.04$
rubber on dry road :	$\mu_s = 1.0$,	$\mu_k = 0.8$
rubber on wet road :	$\mu_s = 0.3$,	$\mu_k = 0.25$

E1 Suppose a box rests on an plane and we push with force F_o at angle θ.



If $\mu_s = 0.1$ is the coeff. of static friction then what is the minimum θ for the box to stay put?



friction acts // to surface.

$$m\vec{a} = \vec{N} + \vec{F}_f + \vec{F}_g + \vec{F}_o$$

$$\underbrace{\begin{aligned} ma_y &= N - mg - F_o \sin\theta \\ ma_x &= F_o \cos\theta - \mu_s N \end{aligned}}_{}$$

<u>max</u> possible $F_{f,\text{static}}$.

to solve these we put $a_x = a_y = 0$ as we want <u>rest</u>

Observe $0 = F_o \cos\theta - \mu_s N \Rightarrow \underline{N = \frac{1}{\mu_s} F_o \cos\theta}$.

then $\underbrace{0 = N - mg - F_o \sin\theta}$

$$N = mg + F_o \sin\theta = \frac{1}{\mu_s} F_o \cos\theta$$

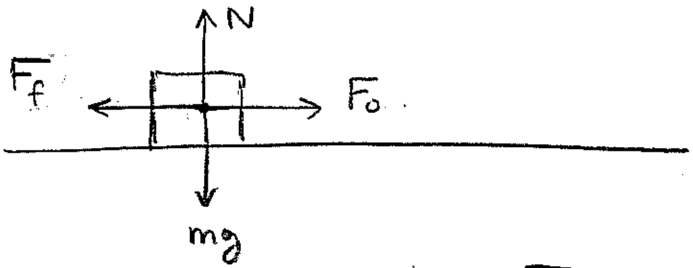
$$\boxed{\frac{mg}{F_o} = \frac{1}{\mu_s}\cos\theta - \sin\theta}$$

for a given mass m, force F_o we can use a numerical method to solve for θ.

<u>SPECIAL CASE</u>: $\theta = 0$ (horizontal force $\vec{F}_o$)

we have $\frac{mg}{F_o} = \frac{1}{\mu_s}$ or $\underline{\mu_s mg = F_o}$.

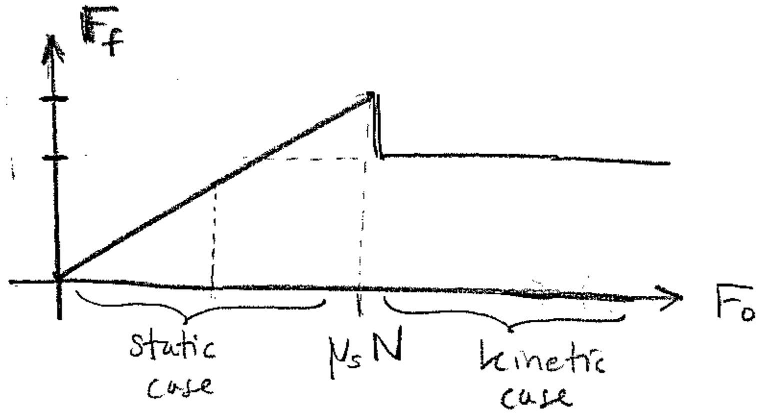
Observation:



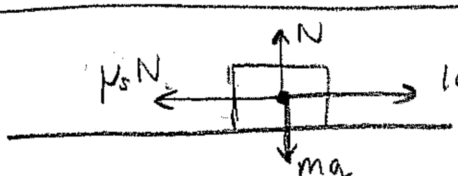
$F_f \le \mu_s N = \mu_s mg$

equilibrium : $F_f = F_o$

We can graph F_f vs. F_o for fixed $N = mg$



E2 Suppose a box has $\mu_s = 0.5$ and $\mu_k = 0.2$ for a given surface and the box just barely starts moving when $F_o = 100N$ is applied horizontally. What is the acceleration if we continue to apply F_o as it slides?



$\Rightarrow$

$$\underbrace{\begin{aligned} 100N - \mu_s N &= 0 \\ N - mg &= 0 \end{aligned}}_{\therefore\ 100N = \mu_s mg}$$

Thus $mg = \dfrac{100N}{\mu_s} = \dfrac{100N}{0.5} = \underline{200N = mg}$.

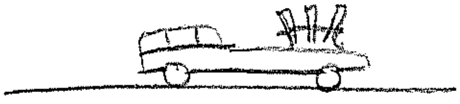
In motion $ma = 100N - \mu_k N = 100N - 0.2(200N)$

$$a = \frac{100N - 0.2(200N)}{m} = \frac{60N}{\frac{200N}{g}} = \boxed{0.3\ g}$$

Comment: as a wheel rolls ideally there is just static friction becauce the wheel is not slipping. At any time the base of tire just lays flat on road. However, in truth, the wheel must be ripped up off pavement as it spins. As this occurs the microscopic hills/valleys of the tire must separate from the microscopic landscape of the road. This creates what is called rolling friction. We typically neglect this since

$$F_{\text{rolling friction}} << F_{f,\text{kinetic or static}}.$$

E3 Suppose you have a car which can produce a force of 1.15 times it weight by its sizable engine. On pavement, should you go full-throttle or dial it back a bit for maximum acceleration?



the F_f on tire/road propels car forward.

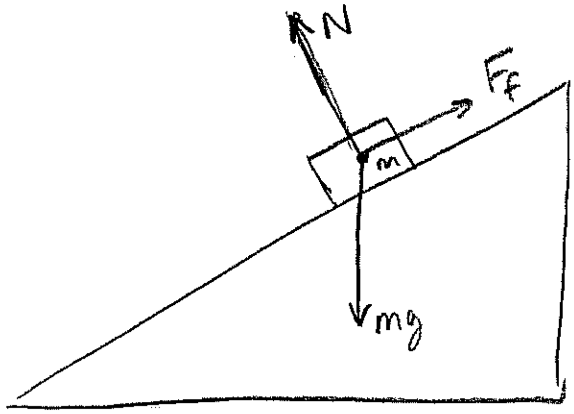
we can produce $F_{f,s} \leq 1.0N$ or $F_{f,k} = 0.8N$

Note $N = mg$ here thus $F_{f,s} = mg$ vs. $F_{f,k} = 0.8mg$

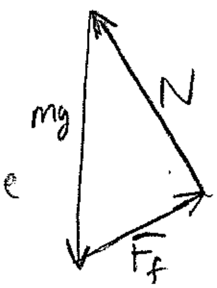
If we spin tires $\Rightarrow F_f = 0.8mg = ma$

whereas if we don't $\Rightarrow F_f = mg = ma.$

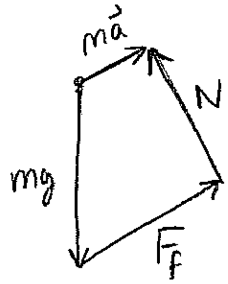
Dial it back a little, mustn't exceed $F_f = mg$.



- For <u>static</u> case we have

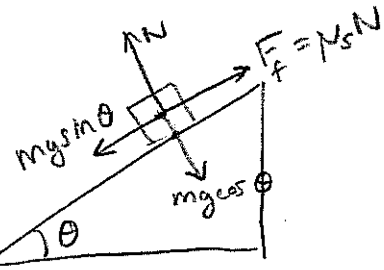


- If $\vec{a} \neq 0$ for m then we have that $m\vec{a} = -mg\hat{j} + \vec{F}_{f,k} + \vec{N}$



when $\vec{a} \longrightarrow \vec{0}$ this shrinks to triangle of static case.

E4



$$N = mg\cos\theta$$

$$\cancel{mg}\sin\theta \leq \mu_s N = \mu_s \cancel{mg}\cos\theta$$

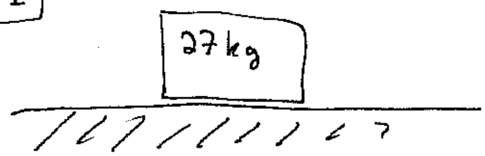
Need $\tan\theta \leq \mu_s$ for static equilibrium.

<u>Note</u>: we can add forces and consider motion. The $\tan\theta \leq \mu_s$ is special formula for special case!

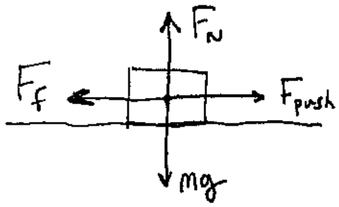
Straight line Motion Friction Examples

(Additional Examples from 2010 WA)

#1



- initially at rest
- 70 N req^{d} to set block in motion.
- 58 N req^{d} to keep block in motion at constant speed.



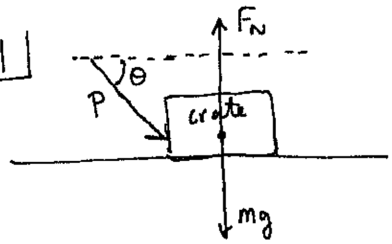
static: $F_f \leq \mu_s F_N = \mu_s mg$

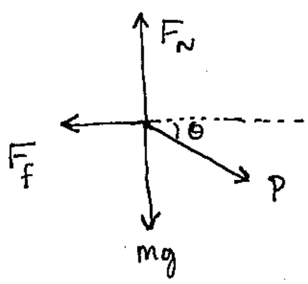
kinetic: $F_f = \mu_k F_N = \mu_k mg$

Thus, $$\mu_s = \frac{70\,N}{(27\,kg)(9.8\,m/s^2)} = 0.265$$

$$\mu_k = \frac{58\,N}{(27\,kg)(9.8\,m/s^2)} = 0.219.$$

#11





For motion we need $F_f < P\cos\theta$ for a nonzero net-force horizontally. Consider then $\mu F_N = P\cos\theta$ for case of motion just starting. Notice

vertical: $0 = F_N - mg - P\sin\theta$

horizontal: $0 = P\cos\theta - \mu F_N$ ($a_x = 0$ for constant velocity)

Note, $F_N = mg + P\sin\theta \Rightarrow \underbrace{0 = P\cos\theta - \mu(mg + P\sin\theta)}_{\text{Solve for } P,}$

$$P = \frac{\mu mg}{\cos\theta - \mu\sin\theta} = \frac{\mu \frac{1}{\cos\theta} mg}{1 - \mu\tan\theta}$$

$$\therefore \boxed{P = \frac{\mu \sec\theta\, F_g}{1 - \mu\tan\theta}}$$

Pulleys & Inclined Planes

#1 (a.) your mass $m = 93\,kg$. What is the acceleration of the earth due to you?

(b.) if you hop 34 cm down then the earth moves through a distance of approx. what distance?

(a) $$\underbrace{M_{EARTH}\, a_{EARTH} = mg}_{\text{implied by } 3^{rd} \text{ Law.}} \longrightarrow a_{EARTH} = \left(\frac{m}{M_{EARTH}}\right) g = \left(\frac{93\,kg}{5.98\times10^{24}\,kg}\right) 9.8\,\frac{m}{s^2}$$

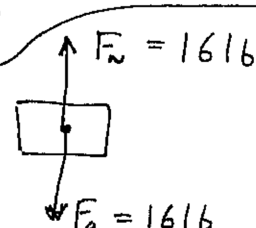
$$\underline{a_{earth} = 1.52\times10^{-22}\,m/s^2}$$

(b.) $\Delta y = \frac{1}{2} a_y (\Delta t)^2$ for $V_{oy} = 0$.

$0.34\,m = \frac{1}{2} g t^2 \longrightarrow t = \sqrt{0.68\,m/g}$

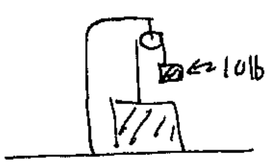
$\hookrightarrow \Delta y = \frac{1}{2}\left(1.52\times10^{-22}\,\frac{m}{s^2}\right)\cdot\frac{0.68\,m}{9.8\,m/s^2} \approx 10^{-24}$

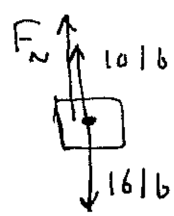
#2 A 16.0 lb block rests on floor.



a.) $F_N = 16\,lb$ (force of floor pushing up, a.k.a the normal force.)

b.) If





$\Rightarrow F_N + 10\,lb = 16\,lb$

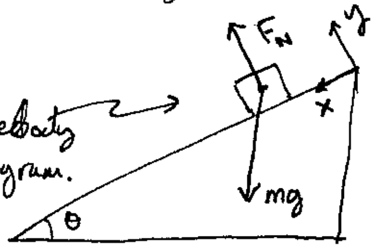
$\therefore \boxed{F_N = 6\,lb}$

c.) if 10 lb is replaced with a 27 lb weight then the block accelerates upward and the normal force vanishes.

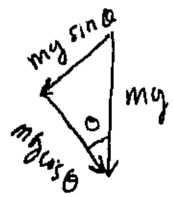
$\underline{F_N = 0}$.

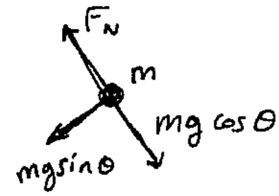
#7 of Pulleys & Planes

freebody diagram.



break down gravity





parallel to plane: $ma_x = mg\sin\theta$

$\perp$ to plane: $ma_y = F_N - mg\cos\theta = 0$

assuming m stays on plane.

We find $F_N = mg\cos\theta$.

Also, $\boxed{a_x = g\sin\theta}$ (down the plane)

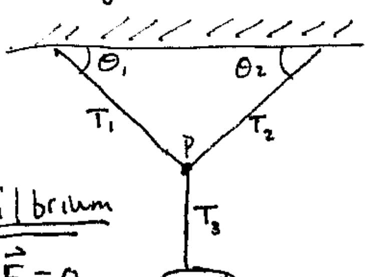
This is a constant acceleration so we can apply the formula $V_f^2 = V_0^2 + 2a_x\Delta X$. The plane has $\Delta X = 2.65m$ (for me)

Thus $V_f = \sqrt{2g\sin\theta\,\Delta X}$ and just plug in $\Delta X = 2.65m$, $V_0 = 0\ m/s$.

Given $\Delta X = 2.65\,m$ and $\theta = 16.9°$ we get $a_x = 2.85\,m/s^2$ and $V_f = 3.89\,m/s$

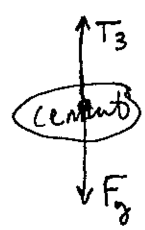
(you had different values so your final answers a bit different)

#3 of Pulleys & Planes

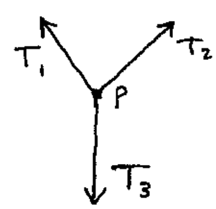


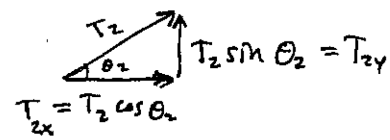
Equilibrium

$\sum \vec{F} = 0$



$T_3 = F_g$





$T_{1x} = T_{2x} \Rightarrow T_1\cos\theta_1 = T_2\cos\theta_2$

$T_3 = T_{1y} + T_{2y} \Rightarrow F_g = T_1\sin\theta_1 + T_2\sin\theta_2$

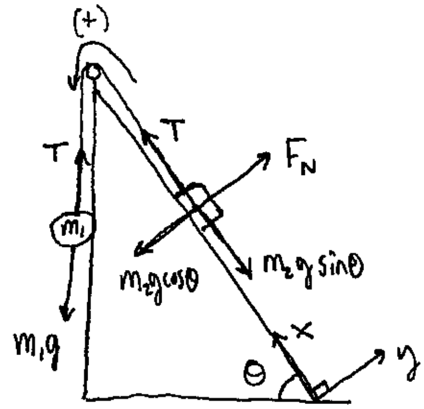
We wish to solve for T_1. Eliminate T_2 by solving for $T_2 = \left(\frac{\cos\theta_1}{\cos\theta_2}\right)T_1$ and substitute into the vertical component of Newton's 2nd Law

$$F_g = T_1\sin\theta_1 + \sin\theta_2\left[\frac{\cos\theta_1}{\cos\theta_2}\right]T_1 = T_1\left(\frac{\sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1}{\cos\theta_2}\right)$$

But, we "know" $\sin(\theta_1+\theta_2) = \sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1$ hence,

$$\boxed{T_1 = \frac{F_g\cos\theta_2}{\sin(\theta_1+\theta_2)}}$$

#10 of pulleys & planes



m_1 & m_2 share same magnitude of acceleration because string does not stretch.

$$m_1 a = m_1 g - T$$

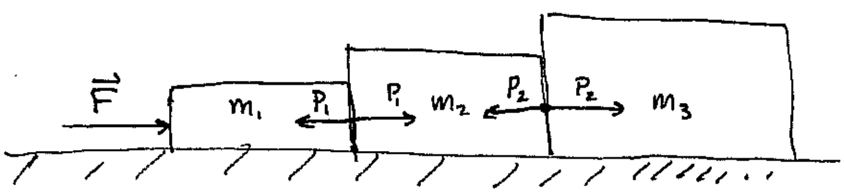
$$m_2 a = T - m_2 g\sin\theta$$

add eq^{n}'s to obtain

$$m_1 a + m_2 a = m_1 g - m_2 g\sin\theta$$

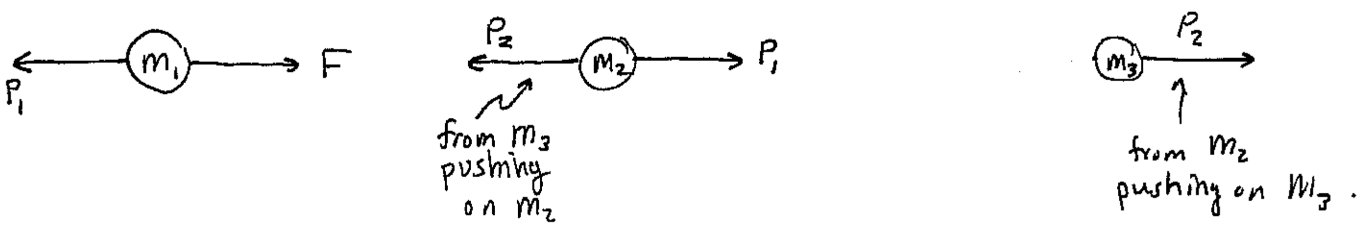
$$\boxed{a = \left(\frac{m_1 - m_2\sin\theta}{m_1 + m_2}\right) g}$$

#17 of pulleys and planes



$a_1 = a_2 = a_3 = a$

they're stuck together.



$$m_1 a = F - P_1$$

$$m_2 a = P_1 - P_2$$

$$m_3 a = P_2$$

$$\underbrace{(m_1 + m_2)a = F - P_2}$$

$$(m_1+m_2)a = F - m_3 a$$

$$\boxed{a = \frac{F}{m_1+m_2+m_3}}$$

(no surprise I hope.)

Hence, $\boxed{P_2 = \dfrac{F m_3}{m_1+m_2+m_3}}$

Also,

$$P_1 = F - m_1 a = F - \frac{m_1 F}{m_1+m_2+m_3}$$

$$= \frac{F(m_1+m_2+m_3-m_1)}{m_1+m_2+m_3} \quad \therefore \quad \boxed{P_1 = \left(\frac{m_2+m_3}{m_1+m_2+m_3}\right) F}$$

If you consider case $m_1 = m_2 = m_3 = m$ we get $P_1 = \frac{2}{3}F$ and $P_2 = \frac{1}{3}F$. nice!