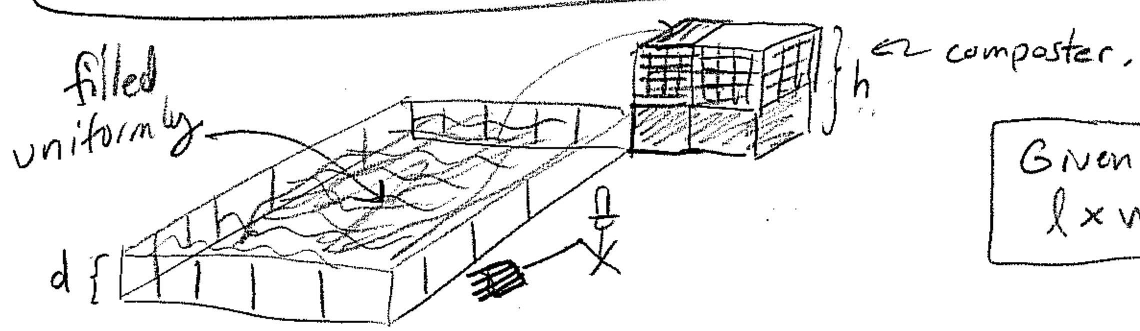
LECTURE 16

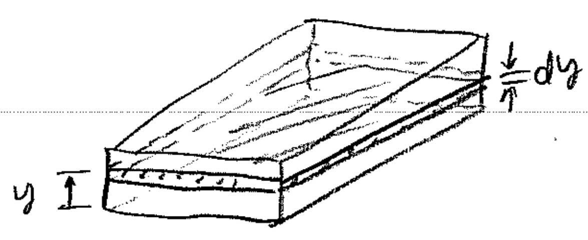
- in this lecture we do work and study energy. An example extending our concept of work to the infinitesimal is given.

E1 Find work done <u>against</u> gravity to shovel a rectangular mess of wet-leafs into a bin which is a height h above the <u>flat</u> ground on which the leafs lay. Pen is $l \times w$



Given pen is $l \times w$

<u>Idea</u>: we cannot just use h because leafs at $y = d$ only have to be lifted $\Delta y = h - d$ whereas leaves at base of pen need $\Delta y = h$. <u>ALL</u> the leaves at a particular y need same Δy to make it up to $y = h$.



Let ρ = density of leafs

$$\rho = \frac{dm}{dV}$$

$$dV = (\text{AREA})\,dy = lw\,dy$$

The mass $dm = \rho\, dV = \rho l w\, dy$ must be lifted $\Delta y = h - y$ against gravity $\Rightarrow dW = (\rho l w\, dy)(y-h)$

$$W = \int_0^d \rho l w (y-h)\,dy = \rho l w \left(\frac{y^2}{2} - hy \right)\Big|_0^d$$

$$= \rho l w \left(\frac{d^2}{2} - hd \right)$$

$$= \rho l w d \left(\frac{d}{2} - h \right)$$

E2 Suppose a particle fixed at the origin exerts a force F_x on a particle with mass m which is proportional to the reciprocal distance 4th power ($F_x = A/x^4$ for some constant A). Assume m is initially resting at $x = x_o$ and find work done by F_x from $x_o \longrightarrow x$

$$W_{x_o \to x} = \int_{x_o}^{x} \frac{A}{\bar{x}^4}\, d\bar{x}$$

$$= \left. \frac{-A}{3\bar{x}^3} \right|_{x_o}^{x}$$

$$= \frac{-A}{3x^3} + \frac{A}{3x_o^3} = \boxed{\frac{A}{3x_o^3} - \frac{A}{3x^3}}$$

It is interesting to find KE and speed of m as $x \to \infty$, By the work-energy Th$^{\underline{m}}$ $W_{x_o \to x} = \Delta KE = \frac{1}{2} m V_f^2$ since $V_o = 0$ was implicitly given by "<u>resting</u>". Thus

$$\frac{1}{2} m V_f^2 = \frac{A}{3x_o^3} - \frac{A}{3x^3} \longrightarrow \frac{A}{3x_o^3} \quad \text{as } x \to \infty$$

Hence $V_f \longrightarrow \sqrt{\dfrac{2A}{3mx_o^3}}$ as $KE \longrightarrow \dfrac{A}{3x_o^3}$ in the limit $x \longrightarrow \infty$. (I assume $x_o > 0$ as does Web Assign in the similar exercise)

<u>Observation</u>: If we consider a path C from some fixed point $\vec{r}_o$ to $\vec{r}(t)$ then $W_{net}(t) = \int_C \vec{F}_{net} \cdot d\vec{r} = K(t) - K(t_o)$ differentiate w.r.t. time t to find

$$\frac{dW_{net}}{dt} = \frac{dK}{dt} = \frac{d}{dt}\left[\frac{1}{2} m \vec{V} \cdot \vec{V}\right] = \frac{1}{2} m \left(\vec{V} \cdot \frac{d\vec{V}}{dt} + \frac{d\vec{V}}{dt} \cdot \vec{V}\right)$$

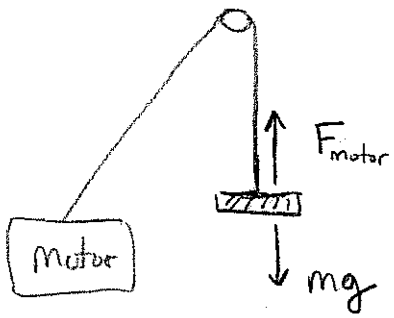
$$= \left(m \frac{d\vec{V}}{dt}\right) \cdot \vec{V}$$

$$= \vec{F}_{net} \cdot \vec{V}$$

The power developed by $\vec{F}_{net}$ is simply $\vec{F}_{net} \cdot \vec{V}$.

E3 Suppose you have a sub-basement filled with robot monkeys. An elevator with mass 31 kg connects the monkeys to the upstairs levels. When in motion the elevator moves 0.32 m/s upward, almost entirely w/o acceleration (ignore the brief start-up motion). The efficiency of the elevator motor is 86%.

If your maximum load for the elevator is 200 kg then what is the minimum power rating you want for the motor?



$$m = 31\,kg + 200\,kg = 231\,kg$$

$$F_{motor} = mg$$ (constant velocity means zero acceleration hence force up must match force down)

By the observation on pg. 2 the motor is delivering a power of $\vec{F}_{motor} \cdot \vec{v} = (mg)(0.32\,m/s)$ since $\vec{F}_{motor} \parallel \vec{v}$,

$$\text{Power Delivered} = (231\,kg)(9.81\,\tfrac{m}{s^2})(0.32\,\tfrac{m}{s})$$

$$= 725.2\ W \qquad W = \frac{J}{s} = \frac{kg\,m^2}{s^3}.$$

Watt Joule/Second

In order for the motor to deliver this much power another 14% needs to be wasted.

$$725.2\,W = (0.86)(\text{Motor Power})$$

$$\boxed{\text{Motor Power} = 843.3\ W}$$