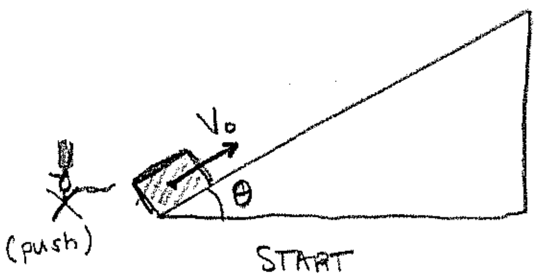
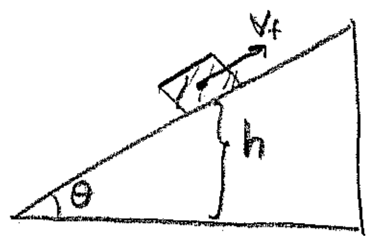
LECTURE 17

- We study energy conservation with & without friction. A generalization of the conservation of energy system is given for systems with partially conservative net-forces.

$\boxed{E1}$ Suppose you push a box up an incline (w/o friction) and give it an initial speed V_0. How high does the box go?

Choose $U = 0$ at base of plane $\Rightarrow$ $U(h) = mgh$ $(h \ll R_{EARTH})$





$$E_0 = \tfrac{1}{2} m V_0^2 + mg(0) \qquad E_f = \tfrac{1}{2} m \underbrace{V_f^2}_{0} + mgh$$

Conservation of energy applies since F_{net} is conservative

$$\therefore \quad E_0 = E_f \Rightarrow \tfrac{1}{2} \cancel{m} V_0^2 = \cancel{m} gh$$

$$\Rightarrow \boxed{h = \frac{V_0^2}{2g}}$$ ← independent of both θ and m.

$\boxed{E2}$ Same as $\boxed{E1}$ except use correct $U(h) = \dfrac{-GmM_E}{(R_E + h)} + \dfrac{GmM_E}{R_E}$

↑ added to make $U(0) = 0$

$$E_0 = E_f \Rightarrow \tfrac{1}{2} \cancel{m} V_0^2 = \frac{G\cancel{m}M_E}{R_E}\left(1 - \frac{R_E}{R_E + h}\right)$$

Solve for h

$$1 - \frac{V_0^2 R_E}{2GM_E} = \frac{R_E}{R_E + h} \quad \therefore \quad \frac{R_E + h}{R_E} = \frac{1}{1 - \frac{V_0^2 R_E}{2GM_E}}$$

$$\hookrightarrow \boxed{h = \frac{R_E}{1 - V_0^2 R_E / 2GM_E} - R_E}$$

challenge show this is $\approx \boxed{E1}$.

What About Friction?

Suppose a mass m has $\vec{F}_{net} = \underset{\text{conservative}}{\vec{F}_{cons.}} + \underset{\text{nonconservative}}{\vec{F}_{n.c.}}$

We have $\vec{F}_{cons.} = -\nabla U$

for some potential energy function U.

$\vec{F}_{n.c.}$ might be friction force. The energy $E(\vec{r},\vec{v}) = \frac{1}{2}mv^2 + U(\vec{r})$ is a function of both position $\vec{r}$ and velocity $\vec{v}$. (could use v)

Consider some path from $\vec{r}_0 = \vec{r}(t_0)$ to $\vec{r}_1 = \vec{r}(t_1)$ say C,

$$W_{net} = \int_C \vec{F}_{net} \cdot d\vec{r}$$

$$= -\int_C \nabla U \cdot d\vec{r} + \int_C \vec{F}_{n.c.} \cdot d\vec{r}$$

$$= -U(\vec{r}_1) + U(\vec{r}_0) + W_{\vec{F}_{n.c.}}$$

But, we know $W_{net} = K_1 - K_0$ from work-energy Th$^{\underline{m}}$.

Thus, $\underbrace{K_1 - K_0 = -U_1 + U_0 + W_{n.c.}}$

$$K_1 + U_1 = K_0 + U_0 + W_{n.c}$$

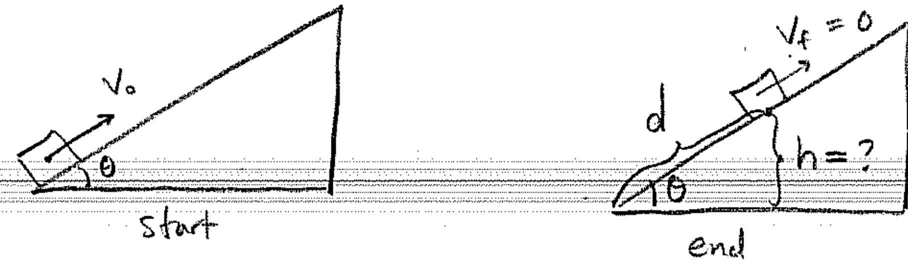
$$\therefore \quad \boxed{E_1 = E_0 + W_{n.c.}}$$

energy is no longer conserved.
It may be added or lost due to the work done by the non conservative force

$W_{n.c.} > 0$ adds energy

$W_{n.c.} < 0$ losing energy

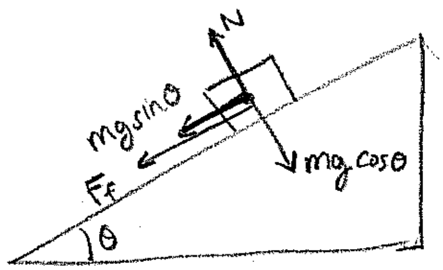
$\boxed{E3}$ repeat $\boxed{E1}$ except add $F_f = \mu_k N$ to the analysis.



$$\sin\theta = \frac{h}{d}$$

$$d = \frac{h}{\sin\theta}$$

What is work done by <u>friction</u>?
Draw freebody diagram to understand



<u>note</u>: $\vec{F}_f$ directed opposite the direction of motion.

$$ma_\perp = N - mg\cos\theta = 0$$

$$\therefore \quad \underline{N = mg\cos\theta}$$

$$\hookrightarrow \quad \underline{F_f = \mu_k mg\cos\theta}.$$

$\vec{F}_f$ is antiparallel to motion and is constant thus

$$W_f = -F_f d = (-\mu_k mg\cos\theta)\left(\frac{h}{\sin\theta}\right)$$

$$E_f = E_o + W_f$$

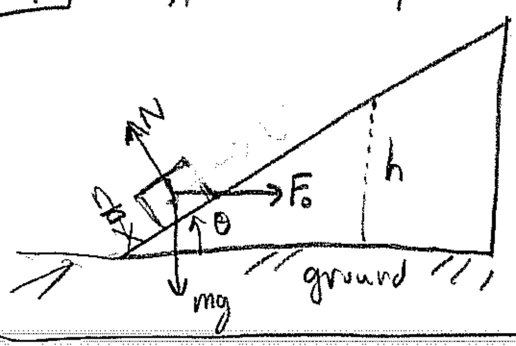
$$\cancel{m}gh = \frac{1}{2}\cancel{m}V_o^2 - \mu_k \cancel{m}g\cot\theta\, h$$

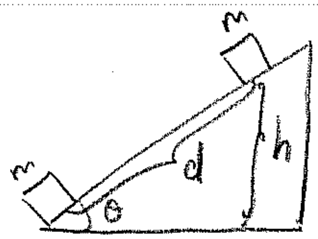
$$h(1 + \mu_k\cot(\theta))g = \frac{1}{2}V_o^2$$

$$\boxed{h = \frac{V_o^2}{2g(1 + \mu_k\cot\theta)}}$$

Note as $\theta \longrightarrow 0^+$ we see $h \longrightarrow 0^+$ as $\cot\theta \longrightarrow \infty$. If $\mu_k = 0$ we recover $\boxed{E1}$ once more.

E4 Suppose pushes with constant force F_0 horizontal to the ground. If he pushes the box to height h then how fast is the box moving? (suppose $V_0 = 0$)

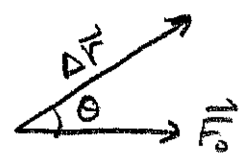




$$\sin\theta = \frac{h}{d}$$

$$\therefore \; d = \frac{h}{\sin\theta}$$

W_T



$$W_{TH} = \vec{F_0} \cdot \Delta\vec{r} = F_0 d \cos\theta$$

Work done by

But, $d = \dfrac{h}{\sin\theta}$ $\therefore$ $\underline{W_{TH} = F_0 h \cot\theta}$

Energy Th$^{\underline{m}}$ says,

$$E_f = E_0 + W_{TH}$$

$$\tfrac{1}{2} m V_f^2 + mgh = \cancelto{0}{\tfrac{1}{2} m V_0^2} + \cancelto{0}{mg(0)} + F_0 h \cot\theta$$

$$m V_f^2 = 2\,(F_0 \cot\theta\, h - mgh)$$

$$V_f^2 = \frac{2\,(F_0 \cot\theta - mg)\,h}{m}$$

$$\boxed{V_f = \sqrt{\frac{2\,(F_0 \cot\theta - mg)\,h}{m}}}$$

Notice $F_0 \cot\theta\, h = F_0 \dfrac{\cos\theta}{\sin\theta} d \sin\theta = F_0 d \cos\theta$
and $mgh = mg\, d \sin\theta$ so we can also write

$$V_f = \sqrt{\frac{2\left[F_0 d \cos\theta - (mg \sin\theta) d\right]}{m}}$$

this is an interesting formula to think about

$F_0 d \cos\theta = W_{TH}$

$-mg \sin\theta\, d = W_{gravity}$