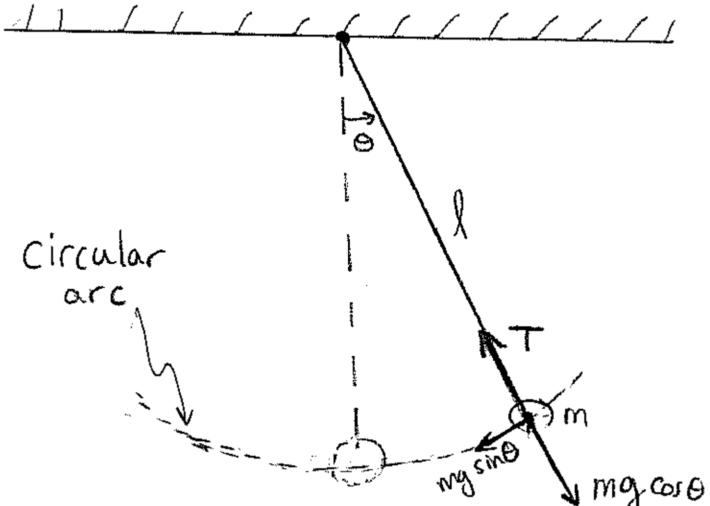
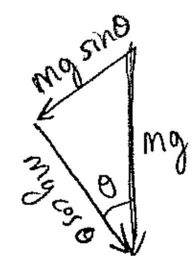
LECTURE 19

- Additional examples on energy and PE analysis are given here. The pendulum is a nice example.

To begin let's examine the simple pendulum:

We attach a mass m to an essentially massless string of length ℓ. Then we imagine the string swings from a frictionless pivot. Apply mechanics to study motion, as usual apply freebody diagram to set-up 2^{nd} Law,





Look at Newton's Eq$^{\underline{n}}$ in the radial and tangential directions,

$$m a_{radial} = mg\cos\theta - T = -mv^2/\ell \qquad \text{(circular motion)}$$

$$\underbrace{m\, a_{tangential} = -mg\sin\theta}$$

$$a_T = -g\sin\theta$$

$$\Rightarrow \boxed{\ell \frac{d^2\theta}{dt^2} = -g\sin\theta} \leftarrow$$ nonlinear DEq$^{\underline{n}}$, exact sol$^{\underline{n}}$ not available, however for <u>small</u> angles $\sin\theta \approx \theta$ and we obtain a simple oscillatory sol$^{\underline{n}}$

$\theta \approx 0$

$$\underbrace{\sin\theta \approx \theta}_{\text{with } 1\%} \Rightarrow \ell\frac{d^2\theta}{dt^2} = -g\theta \rightarrow \boxed{\frac{d^2\theta}{dt^2} + \frac{g}{\ell}\theta = 0}$$

for $|\theta| < 15^\circ$ (see my calculus notes for detailed analysis of this via Taylor's Th$^{\underline{m}}$...)

Continuing: in the small angle approximation we can solve $\frac{d^2\theta}{dt^2} + \frac{g}{\ell}\theta = 0$ in the same way we solved $m\ddot{x} + kx = 0$. The solution is simply

$$\boxed{\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{\ell}}\, t + \phi\right)}$$

This provides a relatively simple method to measure g. Without too much trouble we can easily obtain a value to within 5% or so of the real value.

Remark: the pendulum gives us an example of non constant speed circular motion. At the peak of the swing $v = 0$ and $mg\cos\theta - T = 0 \rightarrow T = mg\cos\theta$. At the bottom of the swing as it passes $\theta = 0$ we reach max. speed v and $T = mg + \frac{mv^2}{\ell}$.
Let's see, ideally if we swing goes to $\theta = 90°$ then we get $T = mg\cos 90° = 0$ ... make sense?

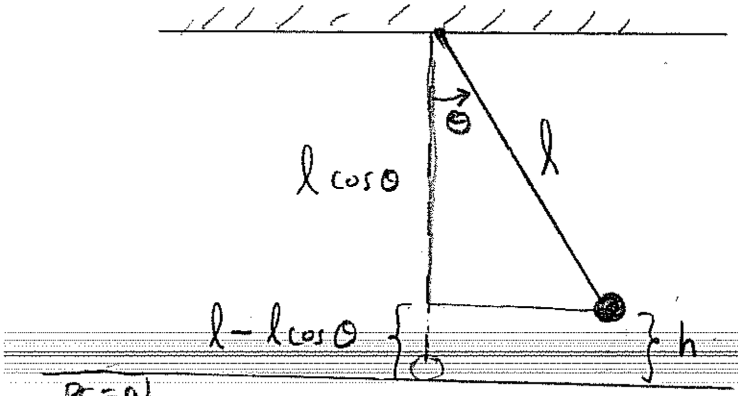
NOTE: $\omega = \sqrt{\frac{g}{\ell}} = 2\pi f = \frac{2\pi}{T}$

$$\therefore \underbrace{\boxed{T = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}}}_{\text{period of pendulum of length } \ell}$$

Maybe $T^2 = \frac{g}{4\pi^2 \ell} \longrightarrow \boxed{g = 4\pi^2 \ell T^2}$

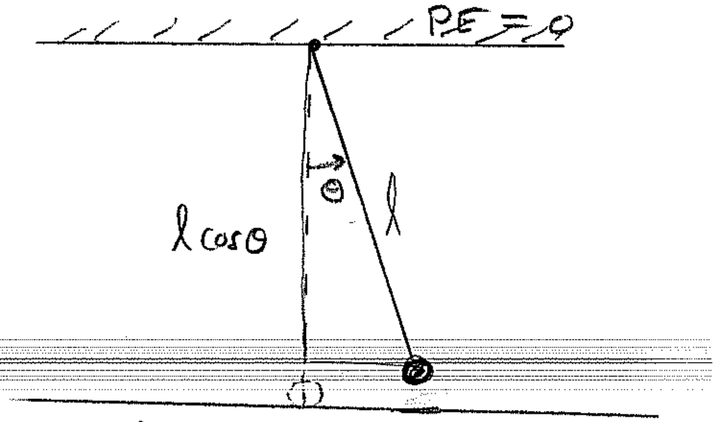
if we measure ℓ & T we get value for g.

Energy Method applied to pendulum problem



If we want PE = 0 at ground then use

$$U = mgh = mgl(1-\cos\theta)$$



If we want PE = 0 at pivot's level,

$$U = -mgl\cos\theta$$

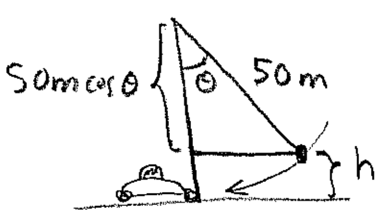
E1 If you want to use a 500 kg wrecking ball to move an old car with mass 1000 kg a distance $d = 2\,m$ across a surface with $\mu_k = 0.3$ then what is the minimum possible angle to swing the ball to accomplish this task. Assume $L = 50\,m$

Idea: the energy of the wrecking ball transfers to the car and gives car sufficient speed to skid $d = 2\,m$ across the surface with $\mu_k = 0.3$

$$E_{\text{lost to friction}} = F_f d = (0.3)(1000\,kg)(9.81\,\tfrac{m}{s^2})(2.0\,m)$$

$$E_{lost} = 5886\ J$$

To give ball $E = 5886\,J$ we need $m_2 g h = 5886\,J$ to begin. Note $h = 50\,m - (\cos\theta) 50\,m$



$$\Rightarrow 5886\ J = m_2 g\,(50\,m - (\cos\theta)50\,m)$$

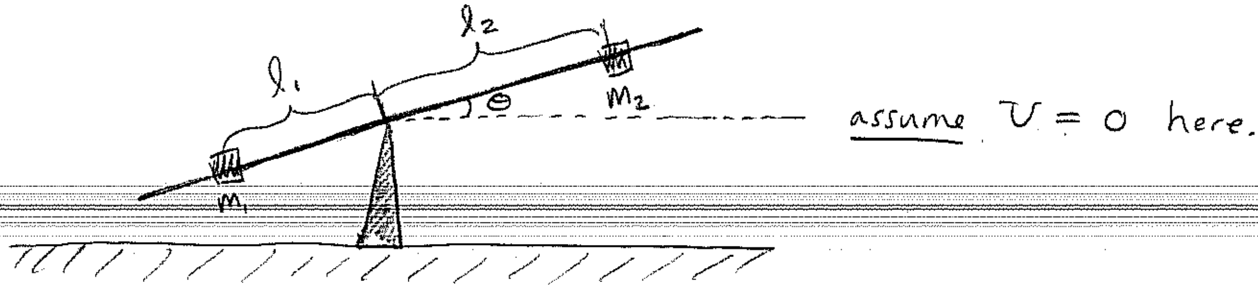
$$\Rightarrow \cos\theta = \frac{(500\,kg)(9.8\,m/s^2)\,50\,m - 5886\,J}{(500\,kg)(9.8\,m/s^2)(50\,m)}$$

$$\cos\theta = 0.975976 \longrightarrow \boxed{\theta = 12.58^\circ}$$

(warning: this example is not accurate ... we'll return later to fix it!)

Additional Examples

E2 Analyze the potential energy of the system pictured below as a function of θ

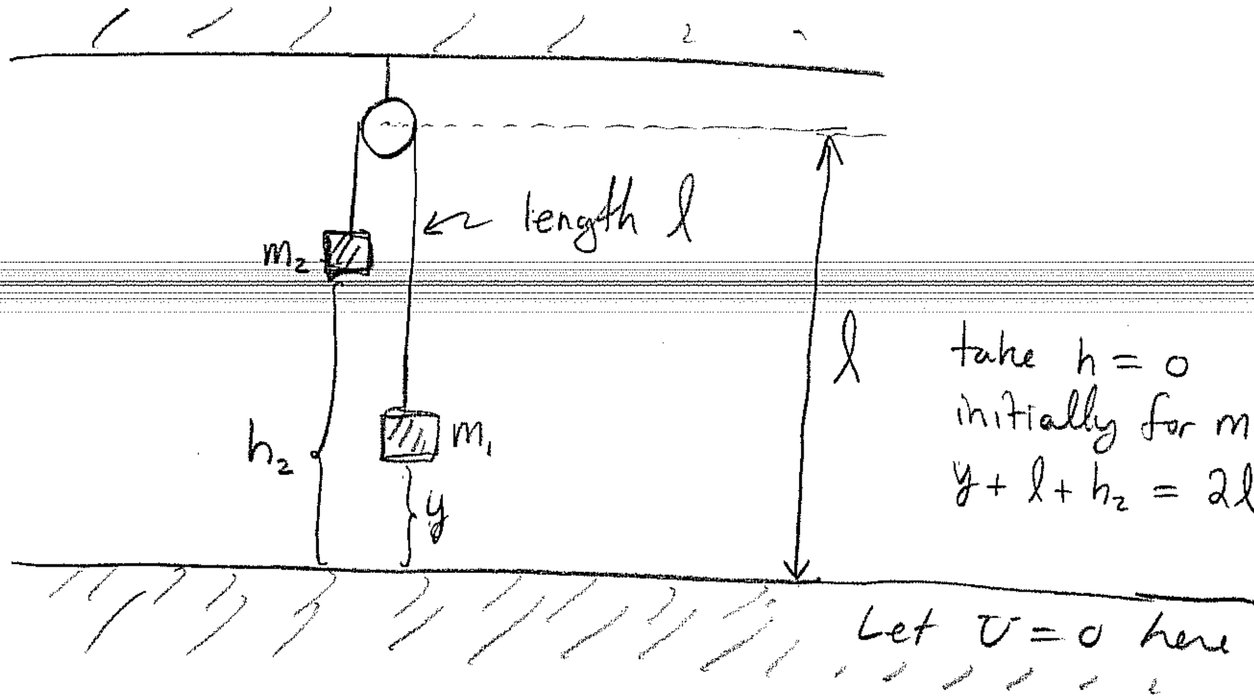


The PE of m_1 and m_2 is determined by h_1 and h_2 the heights of m_1 and m_2 respective. Since we took $U = 0$ at pivot we simply find

$$
\begin{aligned}
U = U_1 + U_2 &= m_1 g h_1 + m_2 g h_2 \\
&= m_1 g(-l_1 \sin\theta) + m_2 g(l_2 \sin\theta) \\
&= g(m_2 l_2 \sin\theta - m_1 l_1 \sin\theta) \\
&= \underline{g(m_2 l_2 - m_1 l_1)\sin\theta}.
\end{aligned}
$$

__Note:__ $\theta = 0 \Rightarrow$ no potential energy stored
$\theta = 90^\circ \Rightarrow$ $m_2 l_2 g$ less $m_1 l_1 g$ PE stored.

$\boxed{E3}$ PE for a pulley two-mass system, discuss ↓



$$U = U_1 + U_2$$
$$= m_1 g y + m_2 g h_2$$
$$= m_1 g y + m_2 g (l - y)$$
$$= m_2 g l + g(m_1 - m_2) y \qquad \text{note: } \underbrace{-\frac{dU}{dy} = (m_2 - m_1) g}_{\text{make sense?}}$$

Now, if $m_2 > m_1$ then $U(y)$ is made smaller as y increases. A system tends to the state of lowest PE so it follows m_2 falls and m_1 rises. (intuitively obvious).

- To solve problems like this you must <u>choose</u> a level for PE = 0 then use

$$U_{1i} + U_{2i} + \frac{1}{2} m_1 V_i^2 + \frac{1}{2} m_2 V_i^2 = U_{1f} + U_{2f} + \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2$$

I leave the details for you to work out.