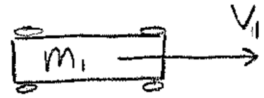
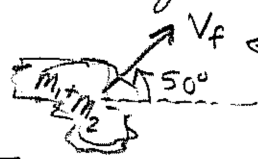
LECTURE 21

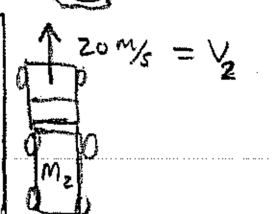
- Momentum for a system of particles or a continuous distribution of mass is conserved if $\vec{F}_{ext} = 0$. We consider a couple examples and introduce the concept of <u>impulse</u> $\left(\Delta \vec{P} = \int_{t_1}^{t_2} \frac{d\vec{P}}{dt}\,dt = \int_{t_1}^{t_2} \vec{F}(t)\,dt\right)$. Finally energy conservation is once more examined and we discuss $E = mc^2$ and energy quantization.

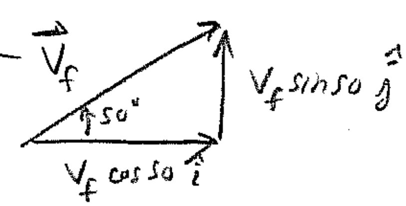
E1) A car with $m_1 = 1000\,kg$ collides with a truck of unknown mass m_2 and the resulting composite mass travels at $50°$ with respect to the path of the car. Supposing the paths were initially $\perp$ and the car had $V_{1A} = 20\,m/s$ whereas $V_{2A} = 40\,m/s$ for truck. Determine the mass of the truck from this data.

We propose m_1, m_2 as a system. The $\vec{F}_{ext} = 0$ provided we examine motion immediately after the collision (otherwise we might include $\vec{F}_{ext} = \vec{F}_{friction}$)









$\vec{V}_1 = (40\,m/s)\,\hat{i}$

$\hookrightarrow \vec{P}_1 = m_1 \vec{V}_1 = \left(40{,}000\,\frac{kg\,m}{s}\right)\hat{i}$

$\vec{V}_2 = m_2\,(20\,m/s)\,\hat{j}$

$\vec{P}_2 = m_2\,(20\,m/s)\,\hat{j}$

$\vec{P}_f = (m_1 + m_2)\,\vec{V}_f$

We have $\vec{F}_{ext} = 0$ hence $\vec{P}_1 + \vec{P}_2 = \vec{P}_f$

$$\hookrightarrow \left(40{,}000\,\frac{kg\,m}{s}\right)\hat{i} + m_2\left(20\,m/s\right)\hat{j} = \left(1000\,kg + m_2\right)\left(V_f \cos 50\,\hat{i} + V_f \sin 50\,\hat{j}\right)$$

continued

E1 continued: study the x, y-components of $\vec{P}_1 + \vec{P}_2 = \vec{P}_f$

$$\hat{i}\,\big|\quad 40{,}000\ \text{kg m/s} = (1000\ \text{kg} + m_2)\, V_f \cos 50^\circ$$
$$\hat{j}\,\big|\quad m_2\,(20\ \text{m/s}) = (1000\ \text{kg} + m_2)\, V_f \sin 50^\circ$$

We find $\tan 50 = \dfrac{(20\ \text{m/s})\, m_2}{40{,}000\ \text{kg m/s}}$

$$\therefore\ m_2 = \left(\frac{40{,}000\ \text{kg}}{20}\right)\tan 50^\circ = \boxed{2383.5\ \text{kg}}$$

We can also find V_f since its interesting,

$$V_f = \frac{40{,}000\ \text{kg m/s}}{3383.5\ \text{kg}\ \cos 50^\circ} = \boxed{18.39\ \text{m/s} = V_f}$$

Remark: the net-KE is $K_1 + K_2 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$ initially then $K_f = \frac{1}{2}(m_1 + m_2) V_f^2$. You can check that $K_1 + K_2 \neq K_f$, this makes the collision <u>inelastic</u>. Energy was <u>lost</u> to the surroundings of the system as the collision occured.

E2) Suppose an avg. force of 100 N propels an object at rest to a speed of 30 m/s. If the mass of the object is 2.0 kg then find the Δt over which the force was applied. Assume

Observe: if $\vec{F} = \vec{F}_o$ (constant) then $\int_{t_1}^{t_2} \frac{d\vec{P}}{dt}\, dt = \int_{t_1}^{t_2} \vec{F}\, dt$ for one-dim'l problem where $\vec{F}$ is in direction x we don't need the vector notation,

$$\Delta P = P(t_2) - \cancel{P(t_1)}^{\,0} = \int_{t_1}^{t_2} F_o\, dt = F_o t\Big|_{t_1}^{t_2} = F_o \Delta t$$

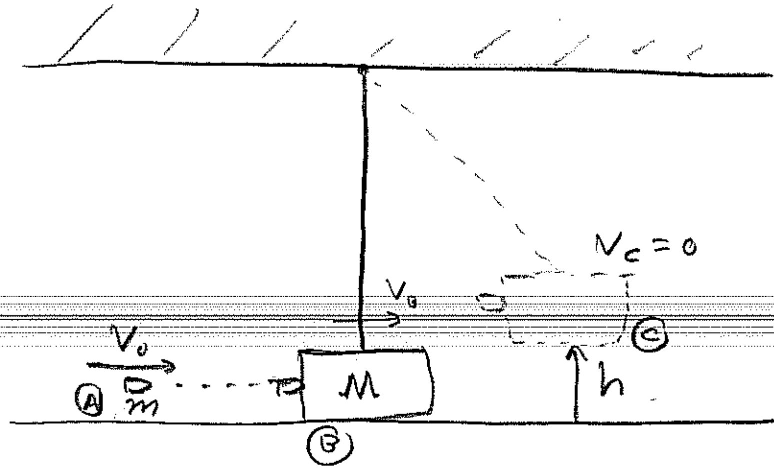
$$(2.0\ \text{kg})(30\ \text{m/s}) = (100\ \text{N})\Delta t$$

$$\Delta t = \frac{60\ \text{kg m/s}}{100\ \text{kg m/s}^2} = \boxed{0.6\ \text{s}}$$

Concept:

$$\boxed{\int_{t_1}^{t_2} \vec{F}(t)\, dt = \text{impulse delivered by } \vec{F} \text{ over } [t_1, t_2] = \Delta \vec{P}}$$

E3 Ballistic Pendulum: when energy is not enough.



we discussed in lecture why energy is not conserved from event A to event B. However going from B to C is a conservative process so we use conservation of E there.

Conserve Momentum from A to B

$$mV_0 = (m+M)V_B$$

$$V_B = \frac{mV_0}{m+M}$$

Next conserve energy from B to C

$$\frac{1}{2}(m+M)V_B^2 = (m+M)gh$$

$$h = \frac{V_B^2}{2g} = \frac{1}{2g}\left(\frac{mV_0}{m+M}\right)^2$$

$$\boxed{h = \frac{m^2V_0^2}{2g(m+M)^2}}$$

Def^{n}/ A Collision is elastic if $KE_i = KE_f$ for system. The Ballistic Pendulum is inelastic because $KE_i \neq KE_f$.

Mass / Energy Equivalence

In the early part of the 20th century a deep error of Newtonian Mechanics was discovered. The idea of absolute time, unlimited velocity and Galilean inertial frames are not physical for large velocities. Einstein's special theory of relativity takes as axioms the constant speed of light, the universality of physics across all frames of reference. These axioms have seeming bizarre consequences but by now special relativity is experimentally verified science. The total mechanical energy of a free particle is given by

$$E = m_0 \gamma c^2$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Binomial Expansion $\longrightarrow$

$$= (1 - v^2/c^2)^{-1/2}$$

$$= 1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots$$

Thus,

$$E = m_0 c^2 \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots\right)$$

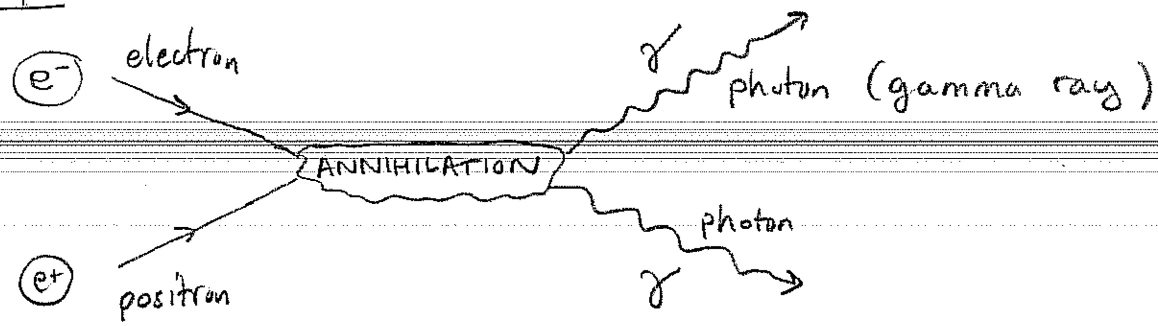
$$= \underbrace{m_0 c^2}_{\text{REST ENERGY}} + \underbrace{\frac{1}{2} m_0 v^2}_{\text{KE IN NEWTONIAN MECHANICS}} + \cdots$$

hidden in classical mechanics usually this stays invariant under non-relativistic processes.

our problems involve this primarily. Since $|v| << c$ the $+ \cdots$ terms are neglible.

CONCEPT: Mass can be created or destroyed and converted to our from energy.

Example:



To treat this properly we must conserve both relativistic momentum and energy. To do this properly we need 4-vectors and Minkowski Space techniques.

Rest Energy of e^- = 0.512 MeV
Rest Energy of e^+ = 0.512 MeV

} property of electrons and positrons (anti-particle to electron)

$$\text{Energy of Photon} = hf$$

h: Plank's Constant
f: frequency of light which is a wave

CONCEPT: For a molecule, or more interestingly a nucleus the net-energy is the sum of rest energy and a rather complicated PE associated with Nuclear forces... Net-result, mass of nucleus $< \sum$ mass of nucleons the extra energy is stored in the binding energy of the nucleus. See Tipler for examples.

Energy Diagrams (useful for gravitation motion chemistry, solid state physics, ... important concept!) ⑥

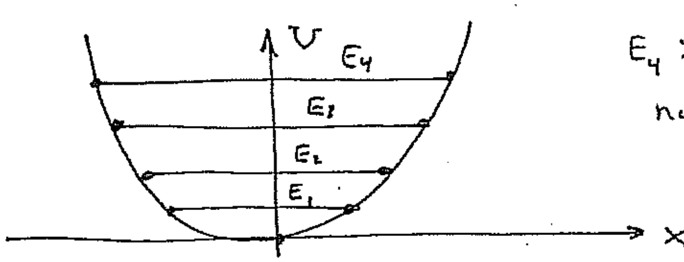
Given a graph of the potential energy function verses x for a one-dimensional system we can easily extract much data about the possible motions of the system.

I'll assume energy $E = K + U$ is conserved, but this discussion is easily twisted to the nonconservative case. The ~~ex~~ crucial observations are as follows:

$$F = -\frac{dU}{dx}$$ ← can see direction of force from slope of U vs. x graph

$$K = \frac{1}{2}mv^2 \geq 0$$ thus $E = E_0$ is not allowed to have $U < E_0$ since $E = K + U \geq U$

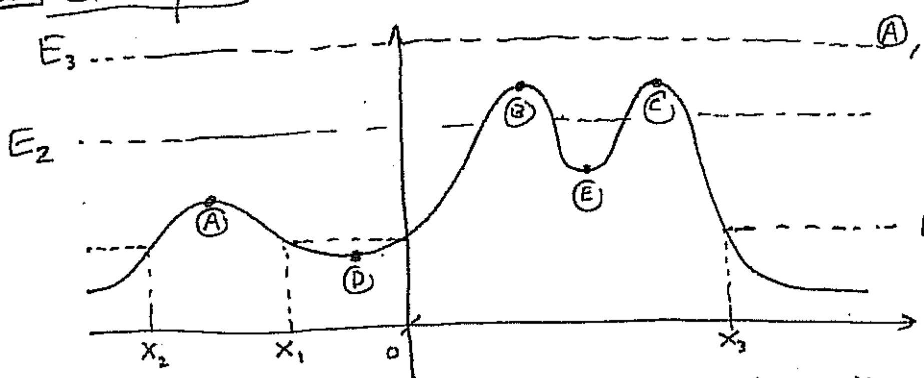
$\boxed{E4}$ Example $U = \frac{1}{2}kx^2$



$E_4 > E_3 > E_2 > E_1$

note, $K = 0$ where the lines intersect $U = \frac{1}{2}kx^2$.

$\boxed{E5}$ Example (Discuss)



Ⓐ, Ⓑ, Ⓒ unstable equilibriums.

Ⓓ, Ⓔ stable equilibriums.

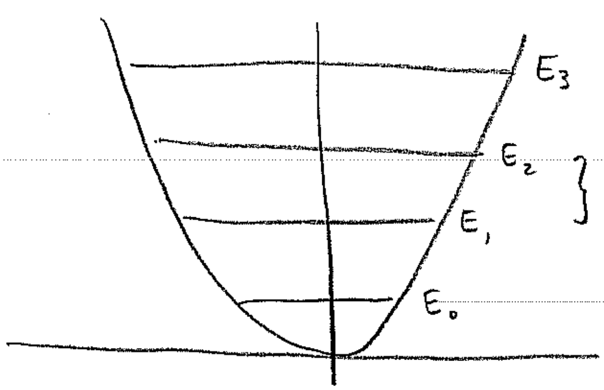
If we have total energy E_1 then either $x \leq x_2$, or $x_1 \leq x \leq 0$ or $x_3 \leq x$. However, $0 \leq x \leq x_3$ is not classically permitted.

(Page 6 was a refresher on Energy Diagrams) ⑦

Energy Quantization

The other major revolution in early 20th century physics was Quantum Mechanics. In a nutshell matter is both a wave & a particle. When waves are stuck between boundaries only certain modes are possible. In some sense electrons orbiting a nucleus are much the same. Only certain orbits are allowed by Quantum Mechanics. Energy is found in discrete packets called quanta for quantum mechanical systems.

$\boxed{E6}$ $E_n = \left(n + \frac{1}{2}\right) hf$ ← energy levels for quantum harmonic oscillator



} very close, can't observe in macroscopic process. Could be ≈ eV for chemical process.