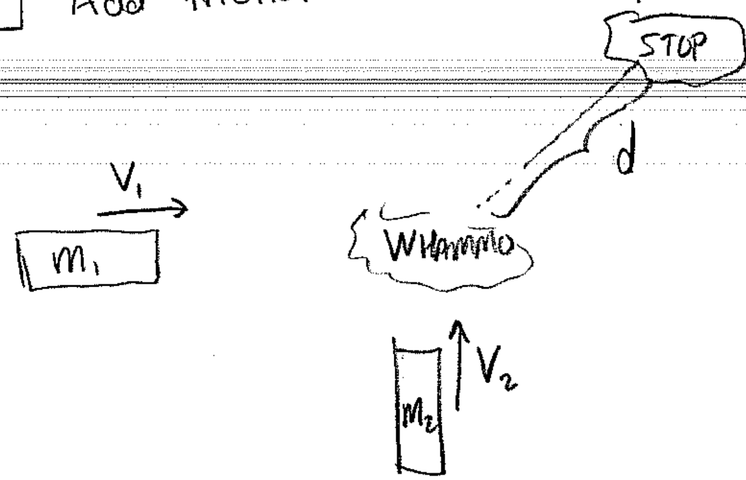
LECTURE 22

- We continue our study of collisions. Some general results for elastic collisions in one dimension are considered. Generalization to 3D slightly ~~out of reach...~~

$\boxed{E1}$ Add friction to collision problem from LECTURE 21



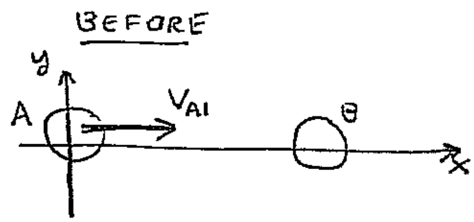
Suppose we have m_1 with V_1 and m_2 with V_2. If the collided mess skids distance d at angle θ and is slowed by $F_f = \mu_k N$ then what physical equations describe this process?

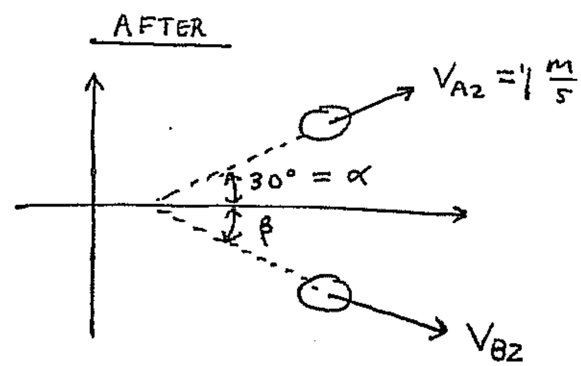
? given in lecture you can add from your notes ☺

Momentum & Collisions

E2

Example: two students on spring break in Canada are sitting on an ice pond in lawn chairs. One of them has an umbrella and a strong wind sends student (A) on a collision course with student (B). Supposing $m_A = 5\,kg$ and $\vec{V}_{A1} = (2\,m/s)\,\hat{i}$ (after the wind sets (A) in motion) if $m_B = 3\,kg$ is initially at rest and after the collision $V_{A2} = 1\,m/s$ at $\alpha = 30°$ then what is $\vec{V}_{B2}$?





Sol^{n}: conserve momentum.

$$\vec{P}_1 = m_A \vec{V}_{A1} = \vec{P}_2 = m_A \vec{V}_{A2} + m_B \vec{V}_{B2}$$

Break it down into components.

$$\vec{V}_{A2} = (\cos 30°)\,V_{A2}\,\hat{i} + (\sin 30°)\,V_{A2}\,\hat{j}$$

$$\vec{V}_{B2} = (\cos\beta\,V_{B2})\,\hat{i} - (\sin\beta\,V_{B2})\,\hat{j}$$

This gives us,

$$\hat{i}: \quad m_A V_{A1} = m_A \cos 30°\,V_{A2} + m_B \cos\beta\,V_{B2}$$

$$\hat{j}: \quad 0 = m_A \sin 30°\,V_{A2} - m_B \sin\beta\,V_{B2}$$

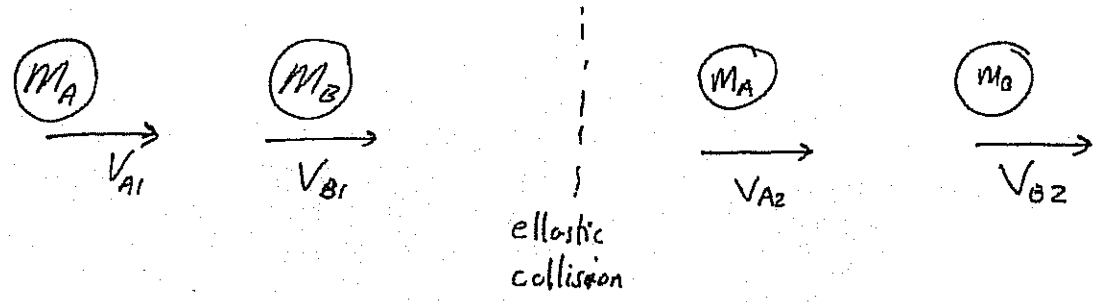
Solve for $V_{B2x} = \cos\beta\,V_{B2} = \dfrac{m_A V_{A1} - m_A \cos 30°\,V_{A2}}{m_B} = 1.89\,m/s.$

Likewise $V_{B2y} = -\sin\beta\,V_{B2} = -\dfrac{m_A \sin 30°\,V_{A2}}{m_B} = -0.83\,m/s,$

Thus $\boxed{\vec{V}_{B2} = \langle 1.89\,m/s,\ -0.83\,m/s\rangle}$ this gives magnitude (a.k.a. speed) of $V_{B2} = 2.1\,m/s$ and $\beta = \tan^{-1}\left(\frac{-0.83}{1.89}\right)$

$\beta = -24°$

Ellastic Collisions General Results



~~Generally we have two conservation eq^ns,~~

$$m_A \vec{V}_{A1} + m_B \vec{V}_{B1} = m_A \vec{V}_{A2} + m_B \vec{V}_{B2}$$

$$\frac{1}{2} m_A V_{A1}^2 + \frac{1}{2} m_B V_{B1}^2 = \frac{1}{2} m_A V_{A2}^2 + \frac{1}{2} m_B V_{B2}^2$$

Simple case to study: Suppose B is at rest initially so $V_{B1} = 0$. Discuss the interesting features of the resulting motion (assume 1-dim'l motion)

Algebra!

$$m_A V = m_A V_A + m_B V_B$$

$$\frac{1}{2} m_A V^2 = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$$

(let $V_{A1} = V$
$V_{A2} = V_A$, $V_{B2} = V_B$
and there are 1-dim'l vectors they can be negative to indicate leftward direction)

Can show that: (see pg. ⑥)

$$V_A = \left(\frac{m_A - m_B}{m_A + m_B}\right) V$$

$$V_B = \left(\frac{2 m_A}{m_A + m_B}\right) V$$

where it also can be shown $V_B = V + V_A$ hence

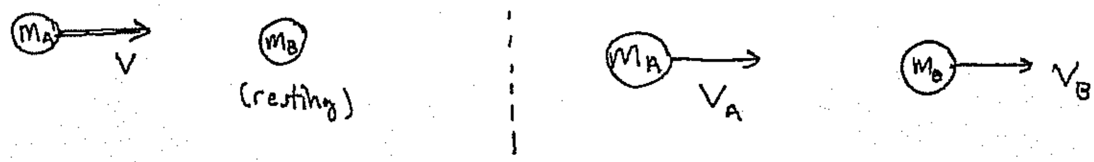
$$\underbrace{V = V_B - V_A}$$

relative velocity opposite before & after collision
since $V_{B1} - V_{A1} = -V$ verse
$V_{B2} - V_{A2} = V_B - V_A = V$.

$$\boxed{\vec{V}_{\text{relative before}} = -\vec{V}_{\text{relative after}} \quad \text{for Ellastic Collisions}}$$

ONE-DIMENSIONAL ELASTIC COLLISION (PROOF)

(BEFORE) (AFTER)



Supposing the collision was elastic yields:

$$\text{I} \quad \frac{1}{2} m_A V^2 = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$$

The system is isolated, so $F_{net} = 0$ and momentum of m_A, m_B is conserved,

$$\text{II} \quad m_A V = m_A V_A + m_B V_B$$

Notice that I yields,

$$V^2 = V_A^2 + \frac{m_B}{m_A} V_B^2 \quad \text{III}$$

I can solve II for V_B to obtain $V_B = \dfrac{m_A (V - V_A)}{m_B}$ IV

Substituting IV into III gives

$$V^2 = V_A^2 + \frac{m_B}{m_A}\left(\frac{m_A (V - V_A)}{m_B}\right)^2$$

$$\Rightarrow V^2 - V_A^2 = \frac{m_A}{m_B}(V - V_A)^2$$

$$\Rightarrow (V - V_A)(V + V_A) = \frac{m_A}{m_B}(V - V_A)^2$$

$$\Rightarrow V + V_A = \frac{m_A}{m_B}(V - V_A)$$

$$\Rightarrow \left(1 - \frac{m_A}{m_B}\right) V = -V_A - \frac{m_A}{m_B} V_A$$

$$\Rightarrow (m_B - m_A) V = -(m_B + m_A) V_A \quad \therefore \quad \boxed{V_A = \left(\frac{m_A - m_B}{m_A + m_B}\right) V} \quad \text{V}$$

Substitute V. into IV to find

$$V_B = \frac{m_A}{m_B}\left(V - \left(\frac{m_A - m_B}{m_A + m_B}\right) V\right) = \frac{m_A}{m_B}\left(\frac{\cancel{m_A} + m_B - \cancel{m_A} + m_B}{m_A + m_B}\right) V = \boxed{\left(\frac{2 m_A}{m_A + m_B}\right) V = V_B}$$

Notice that

$$V_B - V_A = \left(\frac{2 m_A - m_A - m_B}{m_A + m_B}\right) V$$

$$\Rightarrow \boxed{V_B - V_A = -V}$$

Three-dimensional case

Again suppose the collision is elastic.

We have,

$$\text{(I)}\quad \frac{1}{2} m_A \vec{V}_{A1}\cdot\vec{V}_{A1} + \frac{1}{2} m_B \vec{V}_{B1}\cdot\vec{V}_{B1} = \frac{1}{2} m_A \vec{V}_{A2}\cdot\vec{V}_{A2} + \frac{1}{2} m_B \vec{V}_{B2}\cdot\vec{V}_{B2}$$

$$\text{(II)}\quad m_A \vec{V}_{A1} + m_B \vec{V}_{B1} = m_A \vec{V}_{A2} + m_B \vec{V}_{B2}$$

Note (I) yields

$$\vec{V}_{A1}\cdot\vec{V}_{A1} = \vec{V}_{A2}\cdot\vec{V}_{A2} + \frac{m_B}{m_A}\left(\vec{V}_{B2}\cdot\vec{V}_{B2} - \vec{V}_{B1}\cdot\vec{V}_{B1}\right)$$

$$\Rightarrow m_A\left(\vec{V}_{A1}\cdot\vec{V}_{A1} - \vec{V}_{A2}\cdot\vec{V}_{A2}\right) = m_B\left(\vec{V}_{B2}\cdot\vec{V}_{B2} - \vec{V}_{B1}\cdot\vec{V}_{B1}\right)$$

$$\Rightarrow \underline{m_A\left(\vec{V}_{A1} - \vec{V}_{A2}\right)\cdot\left(\vec{V}_{A1} + \vec{V}_{A2}\right) = m_B\left(\vec{V}_{B2} - \vec{V}_{B1}\right)\cdot\left(\vec{V}_{B2} + \vec{V}_{B1}\right)}. \quad \text{(III)}$$

Likewise, (II) yields,

$$\underline{m_A\left(\vec{V}_{A1} - \vec{V}_{A2}\right) = -m_B\left(\vec{V}_{B1} - \vec{V}_{B2}\right)} \quad \text{(IV)}$$

Substitute into (III) to obtain,

$$-m_B\left(\vec{V}_{B1} - \vec{V}_{B2}\right)\cdot\left(\vec{V}_{A1} + \vec{V}_{A2}\right) = m_B\left(\vec{V}_{B2} - \vec{V}_{B1}\right)\cdot\left(\vec{V}_{B2} + \vec{V}_{B1}\right) \quad \text{(V)}$$

jump!

$$\Rightarrow \vec{V}_{A1} + \vec{V}_{A2} = \vec{V}_{B2} + \vec{V}_{B1}$$

$$\Rightarrow \boxed{\vec{V}_{B1} - \vec{V}_{A1} = \vec{V}_{B2} - \vec{V}_{A2}}$$

∴ relative velocity constant under elastic collisions.

<u>Remark</u>: the math has a giant hole in it at the "jump". Generally $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ does not imply $\vec{b} = \vec{c}$. If $\vec{V}_{B1} - \vec{V}_{B2} = \alpha\hat{k}$ then we only learn that $(\vec{V}_{A1} + \vec{V}_{A2})_z = (\vec{V}_{B2} + \vec{V}_{B1})_z$ and no info about the x,y-components is given by (V).

<u>Remark</u>: now, go work out odd problems in Tipler until this is boring!