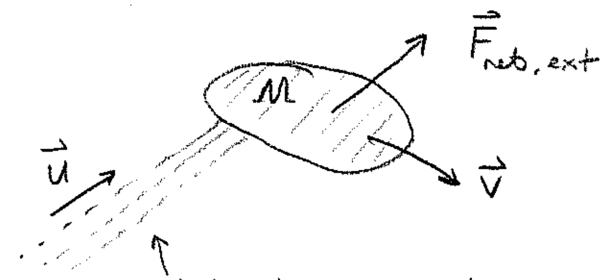
Lecture 24

- We study variable mass problems including rocket flight. (§8.5 of Tipler)

Let's follow the discussion in Tipler,



For small Δt,

$$\vec{F}_{net,ext}\,\Delta t = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$$

$$= (M + \Delta M)(\vec{V} + \Delta\vec{V}) - M\vec{V} - \Delta M\vec{u}$$

$$= M\Delta\vec{V} - \Delta M\underbrace{(\vec{u} - \vec{V})}_{\text{relative velocity of particle stream to } M} + \Delta M\Delta\vec{V}$$

$$\therefore \quad \vec{F}_{net,ext} = M\frac{\Delta\vec{V}}{\Delta t} - \frac{\Delta M}{\Delta t}(\vec{u} - \vec{V}) + \frac{\Delta M\,\Delta\vec{V}}{\Delta t}$$

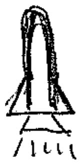
$\Delta t \to 0 \Rightarrow \Delta M, \Delta\vec{V} \to 0$

$\Rightarrow \frac{\Delta M\,\Delta\vec{V}}{\Delta t} \to 0$

$$\boxed{\vec{F}_{net} = M\frac{d\vec{V}}{dt} - \frac{dM}{dt}\vec{V}_{relative}}$$

$$\vec{F}_{net} + \frac{dM}{dt}\vec{V}_{relative} = M\frac{d\vec{V}}{dt}$$

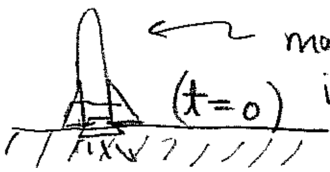
Rocket Problem



$M = M_o - Rt \longrightarrow \underbrace{\frac{dM}{dt} = -R}$

$\leftarrow$ exhaust exiting at $\vec{u}_{exhaust} = \vec{v}_{relative}$

mass decreasing at rate R.



$\leftarrow$ mass M_o includes initial fuel load

$(t=0)$

If we consider the rocket as the system then $\vec{F}_{net,ext} = -Mg\,\hat{j}$ (taking up as positive y).
The generalized Newton's Eq$^{\underline{n}}$ reads:

$$-Mg\,\hat{j} = M\frac{d\vec{V}}{dt} + R\,\vec{u}_{exhaust}$$

Assuming vertical motion and vertical vectoring of the engines so $\vec{u}_{exhaust} = -u_{exhaust}\,\hat{j}$ (downward) we have the one-dim'l problem, (for <u>small</u> altitudes!)

$$-Mg = M\frac{dV_y}{dt} - R\,u_{exhaust}$$

$$\frac{dV_y}{dt} = \frac{R\,u_{exhaust} - Mg}{M} = \frac{R\,u_{exhaust}}{M} - g$$

But, $M(t) = M_o - Rt$ for this simple constant thrust rocket hence,

$$\int_{V_o}^{V_{fy}} dV_y = \int_0^t \left(\frac{R\,u_{ex}}{M_o - R\bar{t}} - g\right) d\bar{t}$$

Continuing

$$V_{fy} = -U_{exhaust} \ln |M_o - R\bar{t}| \Big|_{\bar{t}=0}^{\bar{t}=t} - gt$$

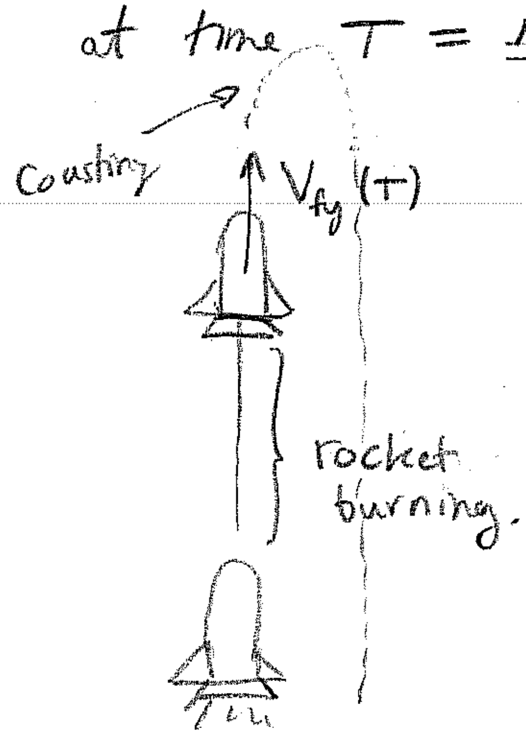
$$V_{fy}(t) = -U_{exhaust} \Big[\ln|M_o - Rt| - \ln|M_o| \Big] - gt$$

$$\boxed{V_{fy}(t) = U_{exhaust} \ln \left| \frac{M_o}{M_o - Rt} \right| - gt}$$

The rocket flies upward only if the term $U_{exhaust} \ln \left| \frac{M_o}{M_o - Rt} \right| > gt$. For this model $V_{fy}(t)$ is only reasonable for t such that $M_o - Rt \geq M_{ROCKET\ UNFUELED}$

$$\left(M_o = M_{ROCKET\ UNFUELED} + M_{FUEL} \right)$$

Once $M = M_{ROCKET\ UNFUELED}$ we're out of fuel at time $T = \dfrac{M_o - M_{ROCKET\ UNFUELED}}{R} = \dfrac{M_{FUEL}}{R}$.



for $t \geq T$ it's a free-fall problem with $M = M_{ROCKET\ UNFUELED}$.

Discussion of Rocket Problem Continued

We found that

$$-Mg = M\frac{dV_y}{dt} - RU_{ex}.$$

$$\hookrightarrow \quad M\frac{dV_y}{dt} = \underbrace{-Mg}_{\text{force of gravity}} + \underbrace{RU_{ex}}_{\text{thrust force}}$$

thrust force denoted F_{th} usually.

E2 Suppose a Saturn V rocket has $M_0 = 2.85\times10^6\ kg$ and $M_{FUEL} = 0.73M_0$, a burn rate $R = 13.84\times10^3\ \frac{kg}{s}$ and $F_{th} = 34\times10^6\ N$. ① What is U_{ex}? ② How long does the rocket burn, ③ what is its initial acceleration?

① $F_{th} = RU_{ex} \Rightarrow U_{ex} = \frac{F_{th}}{R} = \frac{34\times10^6 N}{13.84\times10^3\ kg/s} = \boxed{2.46\ \frac{km}{s}}$.

② $M(t) = 0.27M_0 = M_0 - Rt \Rightarrow t_{burnout} = \frac{-0.73M_0}{R}$

$$\Rightarrow t_{burnout} = \frac{-(0.73)(2.85\times10^6 kg)}{13.84\times10^3\ kg/s} = \boxed{150s}$$

At $t=0$, $M = M_0$ hence,

③ $M_0\frac{dV_y}{dt} = -M_0 g + F_{th}$

$$\frac{dV_y}{dt} = -g + \frac{F_{th}}{M_0} = -g + \frac{34\times10^6 N}{2.85\times10^6 kg}$$

$$= \boxed{2.119\ m/s^2} \approx 21\%\ \text{of } g$$

(like flooring a Toyota into the sky.)

(for practice if you want, send me sol^n for bonus ☺)

$\boxed{E1}$ Suppose you shoot a fire hose at a baby carriage. If the hose shoots 200L of water every minute in a 10cm diameter spray then how fast does the baby go after say 4 seconds (assume the water is captured inside the carriage as time goes on)

Comments on ways to improve Rocket Model

1.) generally $M(t)$ need not be linear. We could have a rocket which turns on and off at various times.

2.) $$F_{gravity} = \frac{-GMM_{EARTH}}{(R_E + y)^2} \approx \underline{Mg} \text{ for } y \approx 0.$$

3.) we could introduce thrust-vectoring so $\vec{u}_{exhaust}$ could change direction of rocket.

4.) if $V_{rocket} \longrightarrow c$ = speed of light then this problem needs to be treated relativistically. (another course ... not here)

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THIS CONCLUDES MATERIAL ON TEST 2

SEE QUIZ 2 FOR YOUR REVIEW.

+10 pts for attendance 3/11/2011