LECTURE 28:

- Further examples of rotational dynamics. Most of the examples here are taken from Tipler's Chapter 9.

E1 A compact disk is rotating at $3000 \frac{rev}{min}$. What is it's angular speed in rad/s? How fast is a point 4cm from the center moving?

$$\omega = \left(3000 \frac{rev}{min}\right)\left(\frac{2\pi \, rad}{rev}\right)\left(\frac{min}{60s}\right) = \boxed{314.16 \frac{rad}{s}}$$

$$V = r\omega = (0.04m)\left(314.16 \frac{rad}{s}\right) = \boxed{12.6 \, m/s}$$

Reminder: $1 \frac{m}{s} = \left(1 \frac{m}{s}\right)\left(\frac{3.281ft}{m}\right)\left(\frac{mile}{5280ft}\right)\left(\frac{3600s}{hr}\right) = 2.237 \text{ mph}$

So, a CD has outer rim moving $\left(12.6 \frac{m}{s}\right)\frac{2.237mph}{m/s} \cong 28mph$.

E2 What is the KE of a rod of length L rotating at angular velocity ω about the axis pictured (mass M)

We know $I = \frac{1}{3}ML^2$ for the rod rotated around the end.

View L as union of $\underbrace{\frac{3}{4}L \text{ with } \frac{3}{4}M}_{I_1}$ and $\underbrace{\frac{1}{4}L \text{ with } \frac{M}{4}}_{I_2}$.

$$\begin{aligned} I_{TOTAL} &= I_1 + I_2 \\ &= \frac{1}{3}\left(\frac{3M}{4}\right)\left(\frac{3L}{4}\right)^2 + \frac{1}{3}\left(\frac{M}{4}\right)\left(\frac{L}{4}\right)^2 \\ &= \frac{27}{12(16)}ML^2 + \frac{1}{12}\left(\frac{1}{16}\right)ML^2 \\ &= \frac{28\,ML^2}{(12)(16)} = \frac{7 \cdot 4}{3 \cdot 4 \cdot 4 \cdot 4}ML^2 = \frac{7}{48}ML^2 \end{aligned}$$

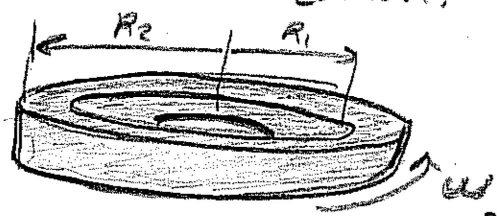
Then $KE = \frac{1}{2}I\omega^2 \Rightarrow \boxed{KE = \frac{7}{96}ML^2\omega^2}$

Remark: there are at least two other half-obvious ways to calculate I for the rod in **E2**.

E3 Suppose a flywheel has a mass of $M = 100kg$ spread uniformly over a hollow cylinder of inner radius $R_1 = 25.0cm$ and outer radius $R_2 = 40.0cm$. Furthermore suppose the flywheel is rev-d up to $\omega = 30{,}000\ \frac{rev}{min}$. How far can you drive a car using this energy if it takes 10.0kW to maintain 40mph (assume you drive at this constant speed.)

We assume 100% efficiency because we've no other info. Of course this is an impossible idealization.

$$KE_i = \frac{1}{2} I \omega^2$$
$$= \frac{1}{4} M (R_1^2 + R_2^2)\, \omega^2$$
$$= \frac{1}{4}(100kg)\left[(0.25m)^2 + (0.4m)^2\right]\left[30000\ \frac{rev}{min}\left(\frac{2\pi\, rad}{rev}\right)\left(\frac{min}{60}\right)\right]^2$$
$$= 5.48 \times 10^7 J$$



A flywheel serves as a type of battery. It stores energy as KE whereas others use chemical, electrical or even gravitational energy. The idea of using a flywheel to power an electric car has been around for decades. Apparently no profitable design exists.

Continuing, $P = \Delta E / \Delta t$ (power is rate of energy change)

$$\Delta t = \frac{\Delta E}{P} = \frac{5.48 \times 10^7 J}{10 \times 10^3 J/s} = 5480 s$$

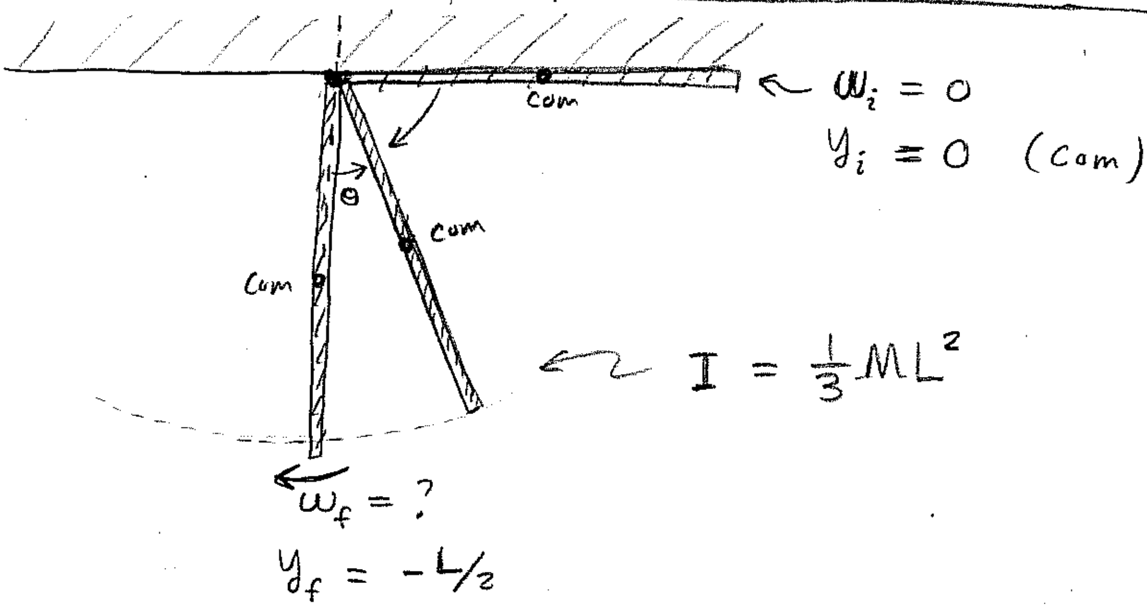
Convert to hrs, $\Delta t = 5480s\left(\frac{hr}{3600s}\right) = 1.52\ hrs.$

$$\Delta x = v \Delta t = (40mph)(1.52hrs) = \boxed{60.9\ miles}$$

(pro-big oil remark ☺, I like my fossil fuels, take that Al Gore!)

Remark: in comparison there are 130 MJ in a gallon of gas. If we assumed 100% efficiency we'd only need about a half gallon of gas to produce the energy of this huge flywheel.

E4 Find the ω_f of a pendulum made of a rod of length L and mass M if let go from $\theta = 90°$



$$E_i = E_f$$

$$\frac{1}{2} I \underbrace{\omega_i^2}_{0} + Mg \underbrace{y_i}_{0} = \frac{1}{2} I \omega_f^2 + Mg y_f$$

$$\omega_f = \sqrt{\frac{-2Mg y_f}{I}}$$

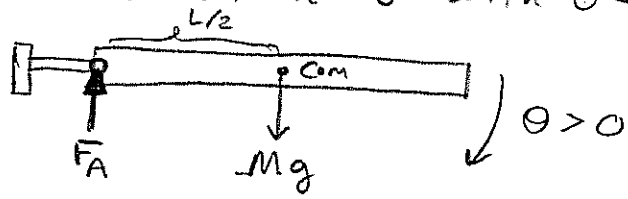
$$\Rightarrow \quad \omega_f = \sqrt{\frac{-2Mg(-L/2)}{\frac{1}{3} M L^2}}$$

$$\boxed{\omega_f = \sqrt{3g/L}}$$

E5 Again consider a rigid rod of length L and mass M. Suppose we release it from a horizontal position and let it drop. Find for $t=\delta$ with $0<\delta<<1$.

a.) angular acceleration of the rod

b.) force F_A exerted on the pivot point



To find the α at $t=\delta$ we need to identify the torques at work, the pivot point and of course I for the system. We've derived that the $\tau_{gravity}$ along the rod is the same as the torque on the Com. Thus,

$$\tau = MgL/2 \qquad (\text{I'm taking CW as } (+) \text{ here})$$

Then we should note F_A does not produce torque since it's at the pivot point. Rod around the endpt. has $I = \frac{1}{3}ML^2$ hence $\tau = I\alpha$ yields,

$$\frac{\cancel{M}gL}{2} = \frac{1}{3}\cancel{M}L^2\alpha \quad \therefore \quad \boxed{\alpha = \frac{3g}{2L}}$$

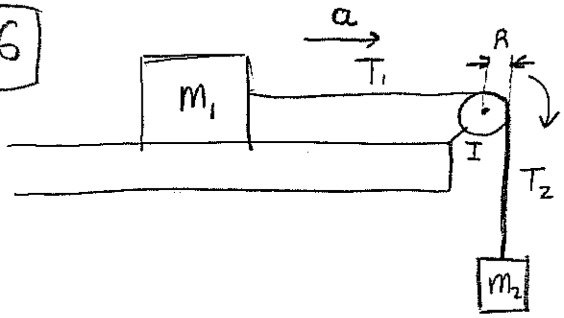
Finding F_A requires a bit more thought. Think about $t=\delta$ the 2^{nd} Law applied to rod says $-F_A + Mg = Ma_T$ [for $t=\delta$ the acceleration radial to the rod is clearly zero since $a_{centripetal} = r\omega^2 = \frac{L}{2}(0)^2 = 0$]

However, $a_T = \frac{L}{2}\alpha$ consequently,

$$\begin{aligned} F_A &= Mg - Ma_T \\ &= Mg - M\frac{L}{2}\left(\frac{3g}{2L}\right) \\ &= Mg - \frac{3}{4}Mg \\ &= \boxed{Mg/4} \end{aligned}$$

Remark: if you're faced with question about force on a pivot you probably need to follow an argument like the one we just did.

E6

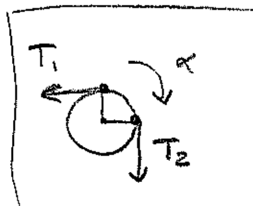


Suppose the blocks pictured are connected by a massive pulley with moment of inertia I and radius R. If the table is frictionless and the rope does not slip find a and α

(1) Newton's 2^{nd} Law on m_2 : $m_2 a = m_2 g - T_2$

(2) 2^{nd} Law on m_1 horizontal : $m_1 a = T_1$

(3) Torque on Pulley (rotational 2^{nd} Law) : $I\alpha = RT_2 - RT_1$



(to check signs)

(4) Nonslip condition : $a = R\alpha$

Combine (3) and (4) to obtain $\underbrace{I\frac{a}{R} = RT_2 - RT_1}$

$$\underbrace{\frac{Ia}{R^2} = T_2 - T_1}_{\star}$$

Add Eq's (1) and (2) to find

$$(m_1 + m_2)a = m_2 g + T_1 - T_2 \overset{\star}{=} m_2 g - \frac{Ia}{R^2}$$

Now solve for a,

$$\left(m_1 + m_2 + I/R^2\right)a = m_2 g$$

$$\therefore \boxed{a = \frac{m_2 g}{m_1 + m_2 + I/R^2}} \Rightarrow \boxed{\alpha = \frac{m_2 g}{m_1 R + m_2 R + I/R}}$$

<u>Remark</u> : If $I = 0$ we recover the earlier result $a = \frac{m_2 g}{m_1 + m_2}$ for a massless pulley. Notice we can <u>not</u> assume $T_1 = T_2$ since the pulley introduces a resistance to motion according to $\tau = I\alpha$. In our earlier treatments the pulley was assumed massless so the tensions were necessarily the same.

E7 Suppose an engine makes a torque of $678\,Nm = \tau$ at $\omega = 4500\ \text{rev}/\text{min}$. Find the power output. How many light bulbs (100W) produce an equivalent energy output?

Idea: $dW = F\,ds = F r\,d\theta = \tau\,d\theta$

$$\hookrightarrow \frac{dW}{dt} = \tau \frac{d\theta}{dt} \Rightarrow \boxed{P = \tau\omega}$$

power developed by torque τ at angular velocity ω.

Applying to E7

$$P = (678\,Nm)\left[4500\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi}{\text{rev}}\right)\left(\frac{\text{min}}{60\,s}\right)\right] = \boxed{319.4\,kW}$$

We'd need $\approx$ 3195 light bulbs.

(it's interesting the same people who refuse to let us grow our power grid want us to turn out lights and get plug-in cars. To scale cars take way more power!)

Comment: Example 9-19 is really neat. Read it.