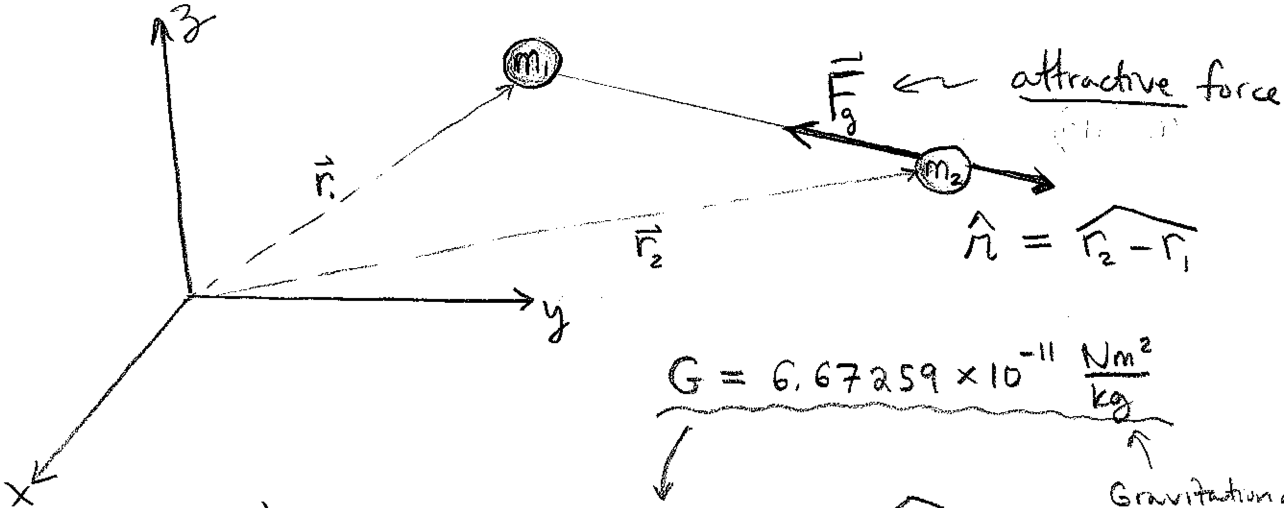
LECTURE 31

- We discuss Kepler's Laws and Newton's Universal Law of Gravitation. These notes contain just a few introductory remarks, links to complete derivations from other courses are provided on website FYI.

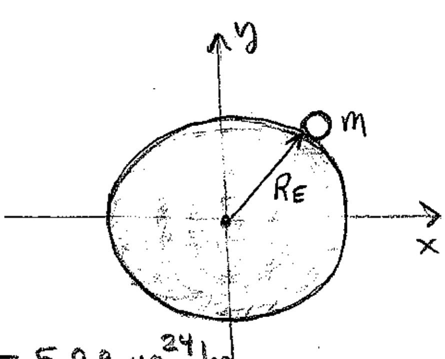


$$G = 6.67259 \times 10^{-11} \frac{Nm^2}{kg}$$

Gravitational constant (not g!)

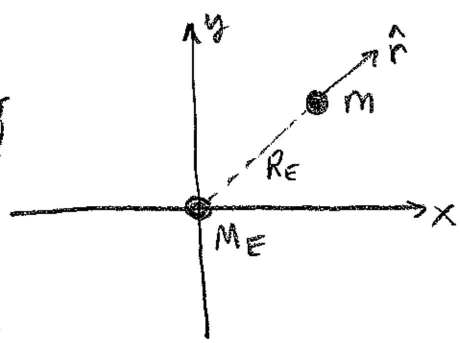
$$\vec{F}_{\substack{\text{gravity of} \\ m_2 \text{ on } m_1}} = -\frac{G m_1 m_2}{\|\vec{r}_2 - \vec{r}_1\|^2} \widehat{\vec{r}_2 - \vec{r}_1} = -\frac{G m_1 m_2}{\mathscr{r}^2} \hat{\mathscr{r}}$$

$\boxed{E1}$ place $m_1 = M_E$ at origin and note: $\vec{\mathscr{r}} = \vec{r} - \vec{0} = \vec{r}$ (place m at $\vec{r}$)



using symmetry of spherical Earth

Center of mass principle.



$M_E = 5.98 \times 10^{24}$ kg

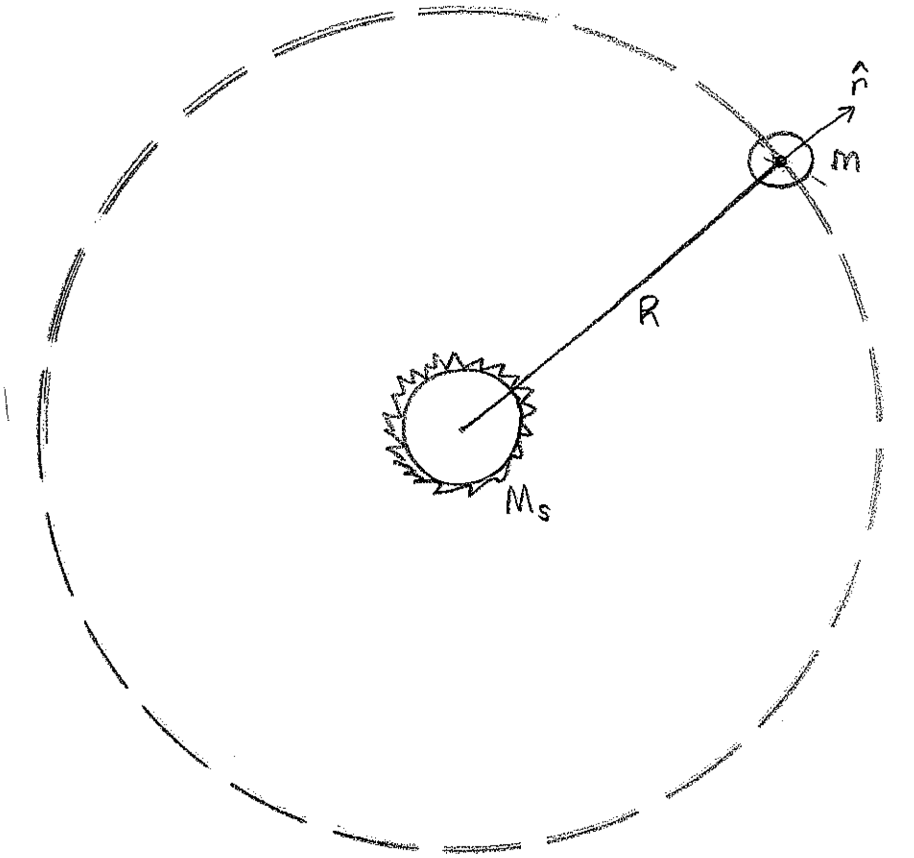
$$F_{g \text{ on } m} = -\frac{G m M_E}{R_E^2} \hat{r} \cong -mg\hat{r}$$

force of gravity on surface of Earth.

(You can calculate $\frac{G M_E}{R_E^2} \approx 9.8 \, m/s^2$.)

Toy derivation of Kepler's Law

Assume a planet with mass m orbits the sun (mass M_s) in a circular orbit. Find how the period T and radius R are related for such an orbit.



Circular Motion: $m\vec{a} = -\dfrac{mv^2}{R}\hat{r}$

Gravity: $\vec{F}_{net} = -\dfrac{GmM_s}{R^2}\hat{r}$

Newton's 2nd Law

$$m\vec{a} = \vec{F}_{net}$$

$$-\frac{mv^2}{R}\hat{r} = -\frac{GmM_s}{R^2}\hat{r}$$

$$\frac{mv^2}{R} = \frac{GmM_s}{R^2}$$

Hence,

$$v^2 = \frac{GM_s}{R} \quad \text{but} \quad v = \frac{2\pi R}{T} \quad \text{(think.)}$$

$$\frac{4\pi^2R^2}{T^2} = \frac{GM_s}{R} \quad \therefore \quad \boxed{T^2 = \left(\frac{4\pi^2}{GM_s}\right)R^3}$$

Remark: the period & radius of an orbit are totally determined by M_s and have nothing to do with the mass of the planet.

Remark: the above remark is too simplistic and these ideas stem from the gross assumptions of the model. In fact M_s moves so the

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whole derivation is suspect. The better derivation works out the motion for the com. One then derives elliptical orbits where both the sun & the planet orbit the com. However, the sun's orbit is small in comparison, so it is a good approximation to suppose the sun is motionless at the center of the orbital plane. I'll post the complete derivation (from a junior-level course we don't offer here at Liberty currently since we don't have a physics Major)

Comment: if we know the R and T then we can judge the mass of the central gravitating object. This sort of logic is employed to suspect the existence of Black Holes. In fact, now Black Holes are an integral component of modern astrophysical theory.