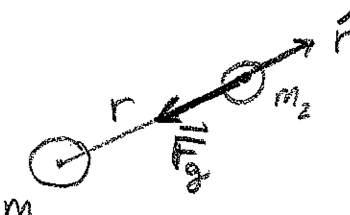
LECTURE 33

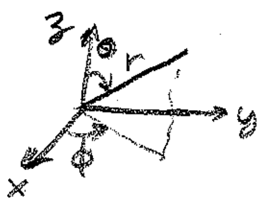
- we worked out the inclined-plane torque problem and discussed bound vs. unbounded orbits as seen from energy analysis. I'll let you get notes on the WA problem from a classmate if you missed it.

Energy Analysis : (place m_1 at origin)

$$\vec{F}_g = -\frac{G m_1 m_2}{r^2}\hat{r}$$



In a good calculus III text (or course) you'll work out that in spherical coordinates (physics spherical meaning of θ and ϕ switched from usual math conventions)



$$\nabla g = \frac{\partial G}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial g}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial g}{\partial \phi}\hat{\varphi}$$

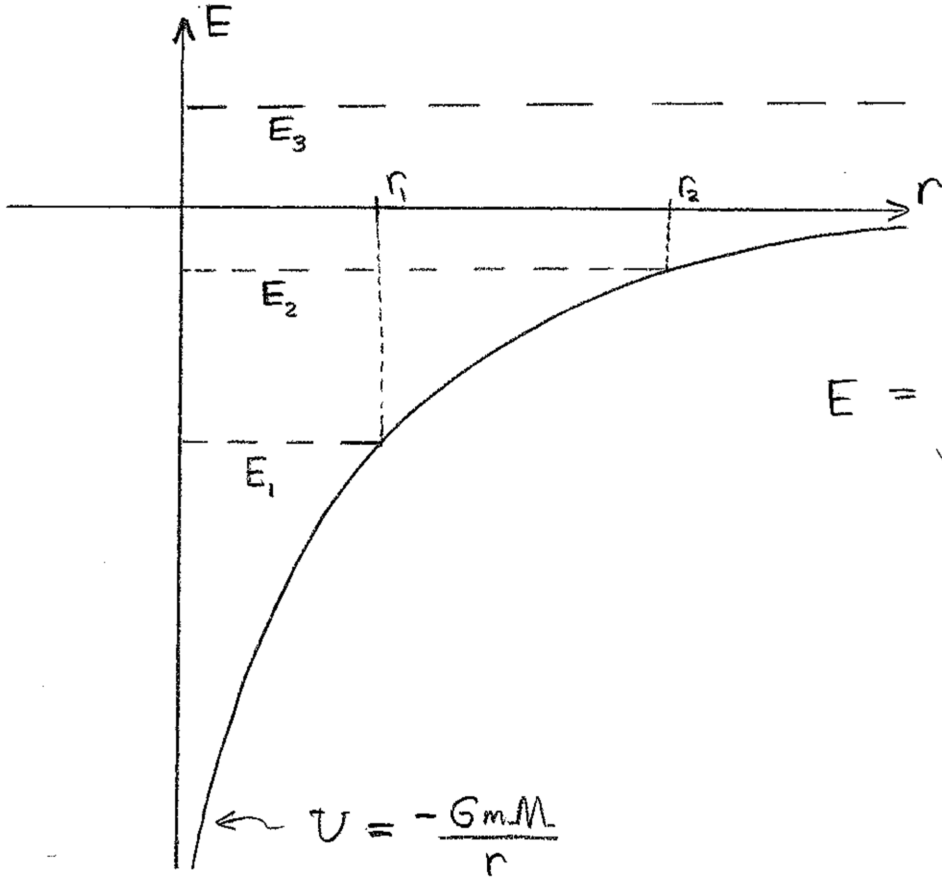
Comparison with $\vec{F}_g$ leads us to conclude a Potential Energy function U with $\vec{F}_g = -\nabla U$ has $\frac{\partial U}{\partial \theta} = 0$ and $\frac{\partial U}{\partial \phi} = 0$ so it only has dependence on the spherical radius $r = \sqrt{x^2+y^2+z^2}$

Moreover, a few moments thought reveals

$$\boxed{U(r) = U_o - \frac{G m_1 m_2}{r}}$$

Often we set $U_o = 0$ which makes $U(r \to \infty) = 0$.

Continuing to analyze orbits



$$\underbrace{E = \frac{1}{2}mv^2 - \frac{GmM}{r}}$$

energy for mass m subject to gravitational field of M

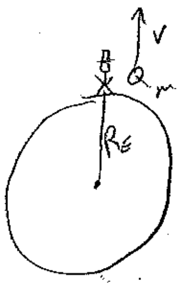
E_1 | The motion is bounded, since $\frac{1}{2}mv^2 \geq 0$ we cannot allow $r > r_1$ since $-\frac{GmM}{r} > E_1$ in that domain. It follows the motion cannot escape $0 \leq r \leq r_1$

E_2 | similar thinking, again bound between $0 \leq r \leq r_2$ since $KE > 0$.

E_3 | since total energy is positive we face no bound on r, as $r \to \infty$ we find $E \to \frac{1}{2}mv^2$.

To complete the story we should think about angular momentum. Many planets travel essentially circular paths... in any event we need $E = 0$ to escape so $E = 0 = \frac{1}{2}\cancel{m}v^2 - \frac{G\cancel{m}M}{R_E} \to \boxed{V_{escape} = \sqrt{\frac{2GM}{R_E}}}$

(for shooting straight up at Earth's surface)



$$\frac{1}{2}\cancel{m}v^2 - \frac{G\cancel{m}M_E}{R_E} = \frac{1}{2}mv_f^2 = 0$$

$$\sqrt{\frac{2GM_E}{R_E}}$$