LECTURE 9

- In this lecture we study Newton's 2^{nd} Law and how to apply it. Vector addition plays important role for two or three dimensional examples.

E1 A mass $m = m_0$ has position at time t

$$\vec{r}(t) = x_0 e^{-\alpha t}\,\hat{i} + \left(y_0 - \tfrac{1}{2}gt^2\right)\hat{j}$$

for some constants x_0, α, y_0 and $g = 9.81\ m/s^2$.
What net-force causes this motion?

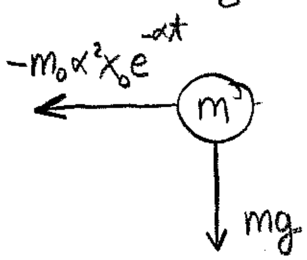
We know $\vec{F}_{net} = m_0\vec{a} = m_0\dfrac{d^2\vec{r}}{dt^2}$. We simply need to twice differentiate $\vec{r}(t)$,

$$\frac{d\vec{r}}{dt} = -\alpha x_0 e^{-\alpha t}\,\hat{i} - gt\,\hat{j} = \underbrace{V_x(t)\,\hat{i} + V_y(t)\,\hat{j}}_{\text{for conversational purposes}}$$

$$\frac{d^2\vec{r}}{dt^2} = \alpha^2 x_0 e^{-\alpha t}\,\hat{i} - g\,\hat{j}$$

$$\therefore \boxed{\vec{F}_{net} = \underbrace{-m_0\alpha^2 x_0 e^{-\alpha t}\,\hat{i}}_{\text{a bizarre frictional force}} - \underbrace{mg\,\hat{j}}_{\text{gravity}}}$$

The freebody diagram would look like:

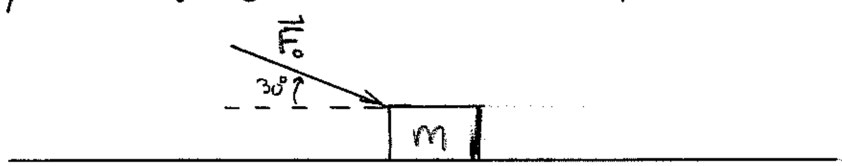


Remark: problems like $\boxed{E1}$ are easier to see through if we omit units. For example,

$$\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k} \longrightarrow \vec{V}(t) = \hat{i} + 2t\hat{j} + 3t^2\hat{k}$$
$$\longrightarrow \vec{a}(t) = 2\hat{j} + 6t\hat{k}$$
$$\longrightarrow \vec{F}_{net} = m(2\hat{j} + 6t\hat{k})$$

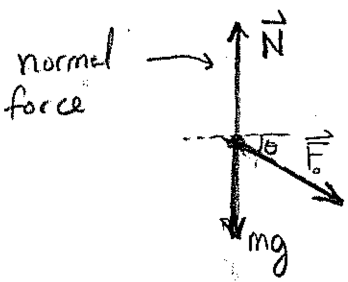
(problem technically is we can't add 2 and 6t because they don't have same units... we ignore this physical objection throughout calculus)

$\boxed{E2}$ Suppose we push a $m = 20kg$ box along a frictionless plane. If the force is applied at $\theta = 30°$ above the horizon results



in an acceleration along the plane of $4 \, m/s^2$ then what is the magnitude F_o ?

To begin we draw the free-body diagram on m which illustrates <u>all</u> the forces which are present,



not to scale, and we idealize the box as a point particle.

$$\vec{F}_{net} = \vec{F}_o + \vec{N} - mg\,\hat{j}$$
$$= F_o\cos\theta\,\hat{i} - F_o\sin\theta\,\hat{j} + N\hat{j} - mg\,\hat{j}$$

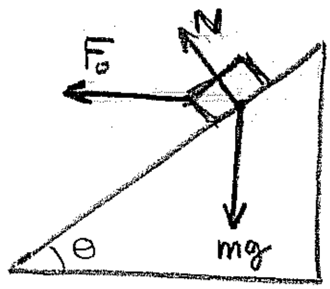
We have $\vec{F}_{net} = m\vec{a} = (20kg)(4\,m/s^2)\,\hat{i}$ thus we find <u>two</u> eq^{n}'s by equating x & y components of $\vec{F}_{net} = m\vec{a}$,

$$\hat{i}: \quad 80N = F_o\cos\theta$$
$$\hat{j}: \quad 0 = -F_o\sin\theta + N - (20kg)(9.81\tfrac{m}{s^2})$$

it turns out we didn't need this for my given question.

Thus $F_o = \dfrac{80N}{\cos 30°} = \boxed{\dfrac{160}{\sqrt{3}}N = F_o}$

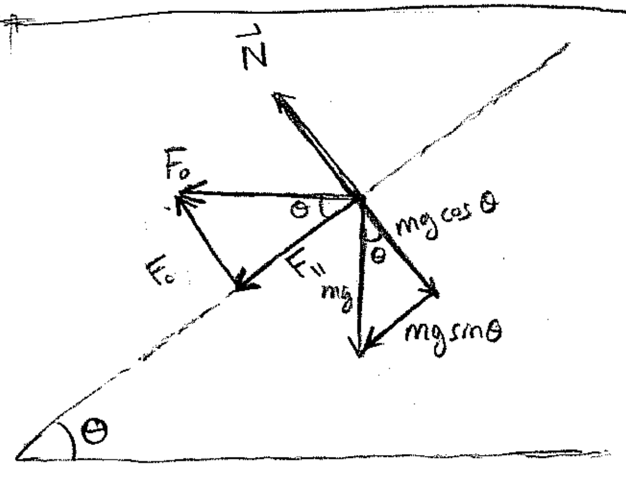
E3 Suppose a force $\vec{F}_0$ pulls a box horizontally as the box slides down an inclined plane with neglible friction. Find the resulting acceleration // to the plane which is at an angle θ

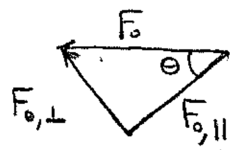


there are three forces at play here.

- the pulling $\vec{F}_0$
- gravity $-mg\hat{j}$
- normal force.

We like to introduce new coordinates relative to the inclined plane. For this problem $\underbrace{a_\perp = 0}_{\text{perpendicular}}$ whereas $\underbrace{a_{\parallel} \neq 0}_{\text{parallel}}$.





$F_{0,\parallel} = F_0 \cos\theta$

$F_{0,\perp} = F_0 \sin\theta$

	$\perp$	$\parallel$
$\vec{N}$	N	0
$-mg\hat{j}$	$-mg\cos\theta$	$mg\sin\theta$
$\vec{F}_0$	$F_0 \sin\theta$	$F_0 \cos\theta$

we'll take down as (+) direction here.

Newton's Law says $\vec{F}_{net} = m\vec{a}$ and $\vec{F}_{net} = \vec{F}_0 + \vec{N} - mg\hat{j}$ hence, as $\vec{a} = a_\perp \hat{u}_\perp + a_{\parallel}\hat{u}_{\parallel}$ we find two eq$^{n's}$:

Continuing: Notice $a_\perp = 0$ since box slides on plane,

$$\left.\begin{aligned} 0 = m a_\perp &= N - mg\cos\theta + F_0 \sin\theta \\ m a_\parallel &= mg\sin\theta + F_0\cos\theta \end{aligned}\right\} \text{together these make } m\vec{a} = \vec{F}_{net}.$$

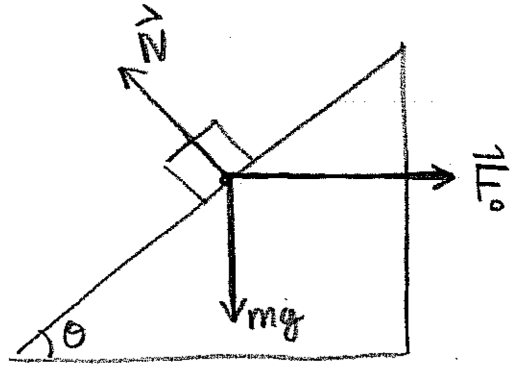
$$\therefore \boxed{a_\parallel = g\sin\theta + (F_0/m)\cos\theta}$$

Limiting Cases?

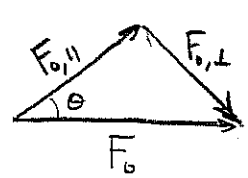
1.) If $F_0 = 0$ and $\theta = 90^\circ$ we get $a_\parallel = g$.
This makes perfect sense. Note, $N = 0$ in this case.

2.) If $\theta = 0^\circ$ then $a_\parallel = F_0/m$
which is also nice. Note, $N = mg$ in this case

$\boxed{E4}$ Draw freebody diagram for $\boxed{E3}$ if $\vec{F}_0$ is pushing instead of pulling.



	$\perp$	$\parallel$
$\vec{N}$	N	0
$\vec{F}_g$	$-mg\cos\theta$	$mg\sin\theta$
$\vec{F}_0$	$-F_0\sin\theta$	$-F_0\cos\theta$



down the plane was (+) in our $\boxed{E3}$ convention