Summary of Convergence and Divergence Tests for Series

TEST	SERIES	CONVERGENCE OR DIVERGENCE	COMMENTS/ THINGS TO WATCH	EXAMPLE
nth-term	$\sum a_n$	Diverges if $\lim_{n\to\infty} a_n \neq 0$	$= 0$ MIGHT converge; need another test	
Geometric Series	$\sum_{n=1}^{\infty} ar^{n-1}$	Diverges if $r \geq 1$ or $r \leq -1$ Converges if $-1 < r < 1$	If converges $S = \frac{a}{1-r}$ Useful with comparison test if the nth term a_n of a series is similar to ar^{n-1}	
P-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Diverges if $p \leq 1$ Converges if $p > 1$	Useful for comparison tests if the nth term a_n of a series is similar to $\frac{1}{n^p}$. Called Harmonic Series when $p = 1$.	
Integral	$\sum_{n=1}^{\infty} a_n$ where $a_n = f(n)$	Diverges if $\int_1^{\infty} f(x)\,dx$ diverges Converges if $\int_1^{\infty} f(x)$ converges	$f(x)$ must be: → positive → continuous → decreasing → able to integrate	
Comparison	$\sum a_n, \sum b_n$ $a_n > 0, b_n > 0$	Diverges: $a_n \geq b_n$ for every n: if $\sum b_n$ diverges, then $\sum a_n$ diverges Converges: $a_n \leq b_n$ for every n: if $\sum b_n$ converges then $\sum a_n$ converges.	$\sum b_n$ is often a geometric series or a p series.	

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Limit Comparison	$\sum a_n, \sum b_n$ $a_n > 0,\ b_n > 0$	If $\lim_{n\to\infty} \frac{a_n}{b_n} = C > 0$, then both series converge or both diverge. C is a finite number.	To find b_n, consider only the terms of a_n that have the greatest effect in the magnitude. If $C = 0$ and b_n converges, then a_n converges.	
Alternating Series	$\sum (-1)^n a_n$ $a_n > 0$	2 conditions: 1. $\lim_{n\to\infty} a_n = 0$ 2. $a_{n+1} \le a_n$	Must Alternate!	
Absolute Convergence	$\sum a_n$ (not alternating series, but some terms negative)	If $\sum \lvert a_n \rvert$ converges, then $\sum a_n$ converges	*Absolute convergence implies convergence *If diverges, might converge-need another test	
Ratio Test	$\sum a_n$	If $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L$ or ∞ then: Diverges if $L>1$ or ∞ Converges if $L<1$ Inconclusive $L=1$	Useful if a_n involves factorials or nth powers. If $a_n > 0$ for every n, the absolute value sign may be disregarded.	