Test	Series	Convergence or divergence	Comments
n^{th} term test for divergence	$\sum_{n=1}^{\infty} a_n$	Diverges if $\lim_{n\to\infty} a_n \neq 0$	Use this test first if divergence is suspected. However, test is inconclusive if $\lim_{n\to\infty} a_n = 0$, so try another test.
Geometric series	$\sum_{n=1}^{\infty} ar^{n-1}$	(a) Converges if $\lvert r \rvert < 1$ (b) Diverges if $\lvert r \rvert \geq 1$	The first term is a, and each term is a multiple, r, of the previous term. Also useful for comparison tests if the n^{th} term of a series is *similar* in magnitude to r^{n-1}. If $\lvert r \rvert < 1$, $s = \frac{a}{1-r}$.
***p*-series**	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	(a) Converges if $p > 1$ (b) Diverges if $p \leq 1$	Useful for comparison tests if the n^{th} term of a series is *similar* in magnitude to $\frac{1}{n^p}$.
Integral Test	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n)$	(a) Converges if $\int_1^{\infty} f(x)dx$ converges (b) Diverges if $\int_1^{\infty} f(x)dx$ diverges	The function f obtained from $a_n = f(n)$ must be **continuous, positive,** and **decreasing**. Use **only** if you can integrate the function.
Individual Comparison Tests not reqd but you can use it you wish	$\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$ $a_n > 0$ and $b_n > 0$	(a) If $\sum_{n=1}^{\infty} b_n$ **converges** and $a_n \leq b_n$ for every n, then $\sum_{n=1}^{\infty} a_n$ **converges.** (b) If $\sum_{n=1}^{\infty} b_n$ **diverges** and $a_n \geq b_n$ for every n, then $\sum_{n=1}^{\infty} a_n$ **diverges.**	The comparison series $\sum_{n=1}^{\infty} b_n$ is often a geometric series or a p-series. To show convergence, you **must** find a series known to converge that is **greater than** the given series. To show divergence, you **must** find a series known to diverge that is **smaller than** the given series. **Hints**: $\ln n < n$ and $\lvert \sin n \rvert$ and $\lvert \cos n \rvert$ are always less than or equal to 1.

Test	Series	Convergence or divergence	Comments
Limit Comparison Tests not req'd but can use if you wish	$\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$ $a_n > 0$ and $b_n > 0$	(a) If $\lim_{n\to\infty} a_n/b_n = L$ for some positive real number L then both series converge or both diverge. (b) If the limit is 0 and b_n is known to converge, then a_n converges, too. (c) If the limit is ∞ and b_n is known to diverge, a_n diverges, too.	To find b_n in the limit comparison test, consider only the terms in a_n that have the greatest effect on the magnitude. Use L'Hopital's rule when needed. This often is helpful when ln (n) appears.
Ratio Test	$\sum_{n=1}^{\infty} a_n$	If $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L$, the series (a) converges (absolutely) if $L < 1$. (b) diverges if $L > 1$ (or ∞)	Inconclusive if $L = 1$. Useful if a_n involves factorials or n^{th} powers. If all terms are positive then absolute value sign may be disregarded. $(n+1)!/n! = n+1$ $2^{n+1}/2^n = 2$ $n+1$ term for $(2n)!$ is $(2(n+1))! = (2n+2)!$
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ $a_n > 0$	Converges if (a) $a_{n+1} \le a_n$ (b) $\lim_{n\to\infty} a_n = 0$	Applies **only** to alternating series. Series diverges if any one of the conditions is not met. Exponent on numeric factor may be n or $n+1$ depending on whether the terms are negative for n odd or even.
Absolute Convergence Test not req'd, but can use if you wish	$\sum_{n=1}^{\infty} a_n$	If $\sum_{n=1}^{\infty} \lvert a_n \rvert$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. If a series converges absolutely, then it converges. A series that converges but does not converge absolutely is called conditionally convergent.	Useful for series containing both positive and negative terms that do not alternate. Use for series with trig functions. This is also useful when you know the non-negative series converges. Adding some negative terms will only make the series converge more quickly. Note the converse to this is not true. For example, the alternating harmonic series converges by the alternating series test but the nonnegative harmonic series diverges.

NOTE: ANY PROBLEM I ASSIGN YOU OUGHT TO BE ABLE TO DO WITH JUST THE req'd TESTS. THE OTHERS ARE USEFUL AS WELL, BUT I'M LIMITING THE SCOPE OF OUR ANALYSIS JUST A BIT.