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Date	Topic	**Notes***	**Assignment**
M: 8-24	Lecture 1: matrix algebra and notation proofs via index calculation	1.1, 1.2, 1.3, 1.4	
W:	Lecture 2: systems of equations over a field, the linear correspondence, structure of solution sets	1.5, 1.6, 1.7	
F:	Lecture 3: Definition of vector space, examples, and subspace test	Chapter 2	
M: 8-31	Lecture 4: linear combinations, span is subspace	Chapter 2	Mission 1
W:	Lecture 5: linear dependence and independence	Chapter 2	
F:	Lecture 6: theory of dimension	Chapter 2	
M: 9-7	Lecture 7: basis and coordinates	Chapter 2	Mission 2
W:	Lecture 8: theory of linear transformations,	Chapter 3	
F:	Lecture 9: rank-nullity theorem, surjective and injective maps	Chapter 3	
M: 9-14	Lecture 10: matrix of a linear transformation	Chapter 3	
W:	Lecture 11: coordinate change & matrix congruence	Chapter 3	Mission 3
F:	Lecture 12: invariant subspaces	Chapter 3	
M: 9-21	Lecture 13: quotient space	Chapter 3	
W:	Lecture 14: first isomorphism theorem	Chapter 3	Mission 4
F:	Lecture 15: first isomorphism theorem	Chapter 3	
M: 9-28	Lecture 16: dual space	Chapter 3	
W:	Lecture 17: dual space	Chapter 3	
F:	Questions		Mission 5
M: 10-5	Boss Fight 1 (covers Lectures 1-17)		
W:	Lecture 18: eigenvectors and eigenspaces, problem of diagonalization	Chapter 4	
F:	Lecture 19: structure of eigenspaces	Chapter 4	
M: 10-12	Lecture 20: Cayley Hamilton Theorem	Chapter 4	
W:	Lecture 21: structure of generalized eigenspaces, theory of Jordan form	Chapter 4	
Fall Break	no class 10-15 & 10-16		
M: 10-19	Lecture 22: Examples of Jordan Form,	Chapter 4	Mission 6
W:	Lecture 23: complexification and the real Jordan form	Chapter 4	
F:	Lecture 24: matrix exponential, properties and applications	Chapter 4	
M: 10-26	Lecture 25: Inner product spaces and norms, orthogonal and unitary operators and matrices	Chapter 5	Mission 7
W:	Lecture 26: Inner product spaces and norms, orthogonal and unitary operators and matrices	Chapter 5	
F:	Lecture 27: Orthogonality, orthonormality, Gram Schmidt algorithm,	Chapter 5	
M: 11-2	Lecture 28: Orthogonal complement, projection, closest vector	Chapter 5	Mission 8
W:	Lecture 29: properties of adjoints and normal operators	Chapter 5	
F:	Lecture 30: spectral theorem & its applications	Chapter 5	
M: 11-9	Lecture 31: bilinear maps, general theory and metrics	Chapter 5	Mission 9
W:	Lecture 32: multilinear maps, general theory and determinants	Chapter 5	

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F:	Lecture 33: metrics and the musical morphism of space and dual space	Chapter 5	
M: 11-16	Questions		Mission 10
W:	Boss Fight 2 (covers Lectures 18-33)		
F:	Lecture 34: antisymmetric tensors and determinants, basis of wedge product	Chapter 6	
	Thanksgiving Break (November 23-27)		
M: 11-30	Lecture 35: associative algebras & exterior algebra of vector space as a formal system	Chapter 6	
W:	Lecture 36: theory of determinants via the wedge product	Chapter 6	
F:	Lecture 37: theory of determinants via the wedge product	Chapter 6	
M: 12-7	Lecture 38: Hodge duality as a musical morphism	Chapter 6	
W:	Questions		Mission 11
F:	Final Boss Fight Part I (covers Lectures 1-37, this is not returned to you in the next class)		
M: 12-14	Final Boss Fight Part II (comprehensive, not the same paper as Part I)		

*these sections are tentative, I may get ahead of what is listed here, it depends on how lecture unfolds. Also, there is some material in the notes which I probably do not cover in this course since we already covered it in depth in Math 221. Also, I'm still working on improving the theoretical development in Chapters 5 and 6, so I will likely add to the notes in lecture. Finally, I have added exercises to the Missions which develop certain ideas before lecture, it is important you do the homework for best understanding. Some proofs are relegated to the Missions.

Grading: usual 1000pts scale with:

Boss Fight 1 = 250pts,

Boss Fight 2 = 250pts,

Missions = 250pts, [2.5pts per problem, I keep the top ten scores]

Final Boss Fight = 250pts.

Remark: The point values indicated above assume that the assignments are taken residentially in the usual classroom or the Testing Center (if you have ODAS accommodations for such). In the case that these are taken in-person, in the usual fashion, it is possible to replace Boss Fight 1 or Boss Fight 2 with the Final Boss Fight grade if helpful. However, in the event the Final Boss Fight cannot be completed residentially either by action of the student or policy of the university then the values of Boss Fights 1 and 2 increase to 370pts each and the Final Boss Fight is worth 10pts and the Mission values increase to make the value of those turned-in residentially to be worth 250pts. Unfortunately, no Boss Fights can be replaced in the event that the Final Boss Fight is not offered as a residentially-proctored exam (for example, if the university was to go "virtual" after Thanksgiving Break then this policy would sadly be enacted).

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Summary of Topic Coverage in Missions 1 – 11:

Each Mission has 10 problems and is worth 25pts.

Problem 1: solve two equations over Z7
Problem 2: count solutions of equation in Z11
Problem 3: LI and spanning in R4
Problem 4: rref of 2x5 and basis for null space, solve related nonhomogeneous problem
Problem 5: elementary matrices
Problem 6: symmetric, nilpotent, antisymmetric
Problem 7: prove distributive rules for scalar and matrix multiplication
Problem 8: product of lower triangular matrices is lower triangular
Problem 9: prove linearity of trace, tr(AB)=tr(BA), invariance of trace under similarity transformation
Problem 10: multiplication by the matrix units, find the center of the nxn matrix algebra
Problem 11: subspaces prove or disprove
Problem 12: modular law for adding subspaces
Problem 13: Cartesian product of vector spaces is vector space
Problem 14: Complexification, obtaining complex vector space from real vector space
Problem 15: vector space valued functions form vector space, linear maps give subspace
Problem 16: direct sum equivalent to unique decomposition of vectors into subspace components
Problem 17: direct sum decomposition of a particular vector space of functions
Problem 18: spanning
Problem 19: linear independence
Problem 20: linear independence of set of derivative maps on space of polynomials
Problem 21: LI elementary lemmas
Problem 22: prove subspace, study dimension and cardinality over real numbers and integers modulo 3
Problem 23: intersection of subspaces and dimension of their sum
Problem 24: extend basis for subspace of polynomials
Problem 25: prove a given subspace is infinite dimensional
Problem 26: on constructing basis for Cartesian product with application to complexification
Problem 27: subspace of symmetric traceless matrices, find its dimension
Problem 28: linear transformation find its matrix and bases for kernel and range, invert the restriction to an appropriate subspace
Problem 29: show linear transformations on polynomials can be surjective and yet not injective.
Problem 30: study of how surjectivity and injectivity of a composite imply the appropriate maps composed are likewise surjective or injective

Problem 31: find basis and construct isomorphism
Problem 32: matrix of linear transformation, trace and determinant to be calculated
Problem 33: illustration of straightening theorem
Problem 34: problem on three independent subspaces
Problem 35: invariant subspaces and the matrix formed as direct sum
Problem 36: invariant subspaces of the symmetrizing map
Problem 37: find bases for three quotient spaces
Problem 38: LI of subset of V implies LI of subset of quotient space
Problem 39: direct sum decomposition for linear transformation which coincides with its square
Problem 40: kernel of composite is inverse image of kernel of the outside map under the inside map
Problem 41: find explicit isomorphism for 5 perhaps interesting examples
Problem 42: prove isomorphism of direct sum decomposed vector space and Cartesian product of quotient spaces
Problem 43: isomorphism of spaces of linear maps
Problem 44: quotient space of real functions modulo even functions
Problem 45: creating inverse map for k-fold derivative maps on the quotient space of kernel
Problem 46: properties of dual basis
Problem 47: trace as dual vector
Problem 48: annihilator and its dimension
Problem 49: the dual of a linear map
Problem 50: given V is direct sum of subspaces, is the dual of V a direct sum of the annihilators of the subspaces ? Prove or disprove.
Problem 51: projection operators derived from direct sum decomposition of finite dimensional vector space
Problem 52: diagonalizable over various fields, inverse as polynomial
Problem 53: decomposition of 3x3 matrix into idempotents
Problem 54: properties of characteristic polynomial
Problem 55: decomposition of vector space w.r.t. linear map whose square is the identity
Problem 56: independence of eigenspaces and diagonalizability of restricted map
Problem 57: diagonalizability for map induced on quotient
Problem 58: linear map on V with polynomial condition is diagonalizable, T/F/Possible
Problem 59: diagonalizable and nilpotent decomposition has commuting components
Problem 60: diagonalizability, decompose space or its complexification into eigenspaces
Problem 61: A,B,C,D are these 3x3 matrices similar ? Find Jordan form to answer.
Problem 62: given characteristic polynomial, find possible Jordan forms associated to A

Problem 63: given Jordan decomposition, construct bases for eigenspace or generalized eigenspace
Problem 64: given ranks of various spaces find Jordan form, characteristic polynomial and minimal polynomial
Problem 65: find real Jordan form for map on span of functions
Problem 66: find real Jordan form for 4x4 matrix, given complex two-chain, use tensor product of matrices for answer
Problem 67: given characteristic polynomial, make table of real Jordan forms, is T or its complexification diagonalizable
Problem 68: Jordan form shifted by multiple of identity
Problem 69: simultaneous diagonalization
Problem 70: cyclic subspaces, some background lemmas for Cayley Hamilton proof
Problem 71: prove identity for determinant of matrix exponential
Problem 72: exponentiation of antisymmetric matrix gives orthogonal transformation
Problem 73: every non-multiple of identity in SO(3) has pair of fixed points, that is it is a rotation about an axis
Problem 74: explain why formulas are not inner product
Problem 75: inner product on polynomials, yet not on continuous functions, why ?
Problem 76: show basis for symmetric matrices as given is Frobenius-orthonormal
Problem 77: find orthonormal basis for span of 1, x and exp(x)
Problem 78: prove or disprove assertions about orthogonal and orthonormal sets
Problem 79: prove Cauchy Schwarz inequality
Problem 80: prove triangle inequality
Problem 81: prove Pythagorean formula extension to inner product space
Problem 82: orthogonal complements of subspaces of inner product space
Problem 83: projection onto subspace of 2x2 matrices
Problem 84: find orthonormal basis to orthogonal complement of polynomial in quadratic polynomial space
Problem 85: find orthonormal basis for orthogonal complement of two point set in n-dimensional space and find closest point.
Problem 86: show orthogonal complement of even functions are odd functions on [-1,1]
Problem 87: calculate adjoint in three cases
Problem 88: Fourier series show and tell
Problem 89: anti-Hermitian map and perpendicularity of its eigenspaces
Problem 90: properties of normal maps TFAE.
Problem 91: find Riesz vector, two real cases, one complex
Problem 92: musical morphism, a bit more background here, Problems 91 and 92 really same
Problem 93: again, find sharp of given dual vectors, exhibiting musical morphism.
Problem 94: Minkowski space and inducing metrics on subspaces

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Problem 95: prove musical morphisms are isomorphisms and that they induce dual metric on the dual space
Problem 96: musical morphism in metric (not an inner product space) context, computationally easy
Problem 97: making new tensor with music, find transformation laws as well
Problem 98: tensor product basis for bilinear maps on the dual space
Problem 99: determinant as the unique completely antisymmetric n-linear map whose value on standard basis is one.
Problem 100: cross product formulas in n=2,3,4.
Problem 101: algebraic basics of exterior algebra
Problem 102: flux and work form maps in three dimensions, cross and dot products
Problem 103: calculate determinant via wedge product
Problem 104: extension of linear map to the exterior algebra and its connection to Laplace expansion by minors
Problem 105: prove the determinant of product is product of determinants
Problem 106: matrix model of exterior algebra on two-dimensional space
Problem 107: prove basic features of the regular representation
Problem 108: find matrix representation of the 3-hyperbolic numbers
Problem 109: find matrix representation of the 2x2 matrix algebra
Problem 110: show Hodge duality for Euclidean space is an isomorphism, calculate Hodge duality on 4-dimensional Euclidean space.