Same instructions as Mission 1. Thanks !

Problem 61 Show that $\sum_{k=1}^{\infty} kz^{k-1}$ converges normally to $g(z) = \dfrac{1}{(1-z)^2}$ on $D = \{z \in \mathbb{C} \mid |z| < 1\}$.

Problem 62 Suppose $f(z) = (1+z)^\alpha$ for $\alpha \in \mathbb{C}$ where we intend to select the principal branch of the α-power function. In other words, we define $f(z) = \exp(\alpha \text{Log}(1+z))$ for $1+z \in \mathbb{C}^-$. Show f is analytic at $z=0$ and the radius of convergence for its power series representation at $z=0$ is $R=1$ independent of α. Determine when the power series truncates to finite order.

Problem 63 Show that if z_o is an isolated singularity of $f(z)$ and if $(z-z_o)^N f(z)$ is bounded near z_o, then z_o is either a removable singularity of $f(z)$ or a pole of order at most N.

Problem 64 Find isolated singularities of the functions below and calculate the principal part of the expansion about each pole, classify the type of each singularity:

(a.) $f(z) = \dfrac{\cos(z)}{z^2 - \pi^2/4}$

(b.) $f(z) = \exp\left(\dfrac{1}{1+z^2}\right)$

(c.) $\dfrac{z}{(z-1)^2(z+1)^2}$

Problem 65 Find all possible Laurent series expansions of $f(z) = \dfrac{1}{z^2 - z}$ centered about $z = 0$. Then find the Laurent series expansion about $z = -1$ which converges at $z = 1/2$.

Problem 66 Find the Laurent series expansions in the following contexts:

(a.) $f(z) = \sinh(z)$ about $z_0 = \pi i$,

(b.) $f(z) = \dfrac{e^z}{(z+1)^2}$ on $0 < |z+1| < \infty$

Problem 67 Find the Laurent series expansions in the following contexts:

(a.) $f(z) = \dfrac{1}{z^2(1-z)}$ for $0 \leq |z| < \infty$

(b.) $f(z) = \dfrac{1}{z^2(1-z)}$ for $|z| > 1$

Problem 68 Multiply series to show $\dfrac{e^z}{z(z^2+1)} = \dfrac{1}{z} + 1 - \dfrac{1}{2}z - \dfrac{5}{6}z^2 + \cdots$ for $0 < |z| < 1$.

Problem 69 Use division of series to derive Laurent series near zero which is centered at zero up to cubic order for:

(a.) $\csc(z)$,

(b.) $\dfrac{1}{e^z - 1}$.

Problem 70 Find partial fractions decompositions for the following:

(a.) $\dfrac{z^3+1}{z^2+1}$

(b.) $\dfrac{z^9+1}{z^6-1}$

Problem 71 Consider $f(z) = \dfrac{z^8}{z^6-1}$. Find the Laurent expansion of $f(z)$ at $z = \infty$. Determine the order of the pole at ∞. Is this function meromorphic on $\mathbb{C} \cup \infty$?

Problem 72 Suppose f has a zero of multiplicity m at $z = a$. Explain why $\dfrac{1}{f}$ has a pole of order m at $z = a$. Apply this result to analyze the poles of $\cot z$ in terms of the zeros of $\tan z$.