Name: (please print name as it appears in Canvas here →) .

MATH 334:	SAM 13: POWER SERIES TECHNIQUE AND SINGULAR POINTS [25PTS]

Box or underline your answer. Work to be shown on separate, single-sided, normal sized paper with no fuzzies. Thanks!

Problem 49 (3pts) Find the first 4 nonzero terms in the power series solution about $x = 0$ for $z'' - x^2 z = 0$.

Problem 50 (4pts) Find the complete power series solution (including a formula for the general coefficient) about $x = 0$ for:

$$y' - 2xy = 0.$$

Problem 51 (4pts) Find the complete power series solution (including a formula for the general coefficient) about $x = 0$ for:

$$y'' - xy' + 4y = 0.$$

Problem 52 (3pts) Find the first four nonzero terms in the power series solution about $x = 0$ for:

$$y'' - e^{2x}y' + (\cos x)y = 0, \qquad y(0) = -1,\ y'(0) = 1.$$

Problem 53 (3pts) Find the first four nonzero terms in the power series solution about $x = 0$ for:

$$z'' + xz' + z = x^2 + 2x + 1.$$

Problem 54 (2pts) Find the singularities of $x(x^2+2x+2)y''+(x^2+1)y'+3y = 0$ and determine the largest open interval of convergence for a solution of the form $y = \sum_{n=0} a_n(x+2)^n$. *Think. Do not try to solve this, I'm asking you about the interval of convergence, I'm* **not** *asking for what a_n are in particular*

Problem 55 (2pts) Suppose we define $e^z = \displaystyle\sum_{n=0}^{\infty} \frac{1}{n!} z^n$. Show that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$.

Problem 56 (2pts) Suppose $\displaystyle\sum_{k=0}^{\infty} (a_{2k}x^{2k} + b_{2k+1}x^{2k+1} = e^x + \cos(x+2)$. Find explicit formulas for a_{2k} and b_{2k+1} via Σ-notation algebra.

Problem 57 (2pts) Find a power series solution to the integrals below:

(a.) $\displaystyle\int \frac{x^3 + x^6}{1 - x^3}\, dx$

(b.) $\displaystyle\int x^8 e^{x^3+2}\, dx$