

Name: (please print name as it appears in Canvas here →) _____.

MATH 334:

SAM 9: CAUCHY EULER AND MORE! [15PTS]

Box or underline your answer. Work to be shown on separate, single-sided, normal sized paper with no fuzzies. Thanks!

Problem 1 Solve $y'' + 3y' + 2y = x + e^{-x} + e^{3x}$.

Problem 2 Solve $y'' + 2y' + y = \frac{e^{-x}}{x+1}$.

Problem 3 Solve $x^2y'' - (x^2 + 2x)y' + (x+2)y = x^3$. Note $y_1 = x$ is a fundamental solution of the DEqn.

Problem 4 Solve the following cauchy euler problems. Give your solution as a real linear combination of the real-value functions in the fundamental solution set.

(a.) $4x^2y'' + y = 0$

(b.) $x^2y'' - 3xy' + 5y = 0$

(c.) $2x^2y'' + 3xy' - y = 0$

(d.) $x^3y''' + 2x^2y'' - xy' + y = 0$

(e.) $x^2y'' + 5xy' + 4y = 0$ with $y(1) = 2$ and $y'(1) = -3$

Problem 5 Solve $(6T^2 - T - 1)[y] = 0$ given that $T = x \frac{d}{dx}$.

Problem 6 Let g be a continuous function. Find an integral solution for $x > 0$ for the Cauchy Euler problem $x^2y'' + xy' + 9y = g$.

Problem 7 A mass $m = 2$ whose motion is along the y direction is pushed by a spring with $k = 4$ and is subject to a velocity dependent friction force of $F_f = -5v$. Find the equation of motion given that $y(0) = 4$ and $v(0) = 0$.

Problem 8 Suppose a mass m is subject to a spring force $F_s = -kx$ and velocity dependent friction force $F_f = -\beta v$ as it moves along the x -direction. Let x_o and v_o be the initial position and velocity of the spring. Derive the formula for potential energy in the spring as a function of time t given that $\beta, k, m > 0$ and $\beta^2 - 4mk > 0$.

Problem 9 Suppose a mass m is subject to a spring force $F_s = -kx$ and velocity dependent friction force $F_f = -\beta v$ as it moves along the x -direction. Let $\alpha > 0$. If an additional force $F(t) = F_o e^{-\alpha t}$ is applied and the mass is initially at the origin with velocity v_o then find the equation of motion given that $\beta^2 - 4mk < 0$.