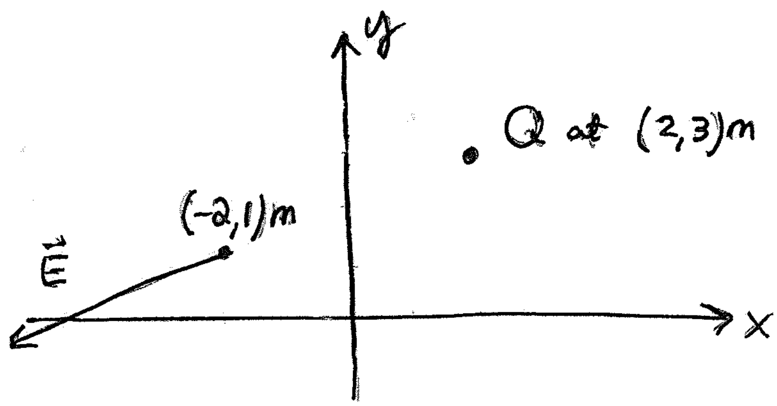
Print Names in Team: ____.

PHYSICS 232	MISSION 1: ELECTRIC FIELDS & COULOMB'S LAW

Please work each problem in the white space provided. Box your answer and explain your steps. Attach additional sheets if necessary. Print this one-sided and staple in the top left corner with a metal staple once complete. Each team turns in one document.

Problem 1 Suppose a charge $Q = 3.0\ nC$ is placed at $(2, 3)m$. What is the magnitude and direction of the electric field due to Q at $(-2, 1)m$? Give the direction of the electric field in terms of the standard angle.



$$\vec{E} = \frac{kQ}{r^3}\vec{r} \qquad \vec{r} = (-2,1)m - (2,3)m = \langle -4,-2 \rangle m$$

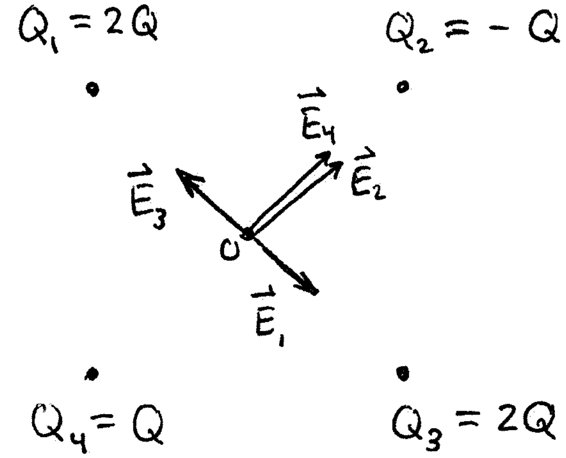
$$= \frac{(9\times10^9)(3\times10^{-9})}{(\sqrt{16+4})^3}\langle -4,-2 \rangle \frac{N}{C}$$

$$= \langle -1.207\frac{N}{C}, -0.604\frac{N}{C} \rangle$$

$$E = \sqrt{(-1.207\frac{N}{C})^2 + (-0.604\frac{N}{C})^2} = \boxed{1.35\frac{N}{C}}$$

$$\boxed{\theta = 206.6^\circ}$$

Problem 2 Suppose charges of $Q, 2Q, -Q$ and $2Q$ are placed at $(-L,-L), (-L,L), (L,L)$ and $(L,-L)$ respectively. Find the electric field at the origin.



At the origin,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

Notice $\vec{E}_1 = -\vec{E}_3$, these cancel out; $\vec{E}_1 + \vec{E}_3 = 0$

Likewise, by symmetry, $\vec{E}_2 = \vec{E}_4$ thus $\vec{E} = 2\vec{E}_2$

$$\vec{E} = \frac{-2kQ}{r^2}\hat{r} \qquad \vec{r} = \langle -L, -L \rangle$$

$$= \frac{-2kQ}{r^3}\langle -L, -L \rangle \quad , \quad r = \sqrt{(-L)^2 + (-L)^2}$$

$$= \frac{2kQ}{(\sqrt{2L^2})^3}\langle L, L \rangle$$

$$= \frac{2kQ}{2\sqrt{2}L^3}\langle L, L \rangle$$

$$= \frac{kQ}{L^2}\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \left\langle \frac{kQ}{\sqrt{2}L^2}, \frac{kQ}{\sqrt{2}L^2} \right\rangle$$

$$\therefore \quad E = \frac{kQ}{L^2} \quad \text{and} \quad \theta = 45^\circ$$

Problem 3 Suppose a charge $Q_1 = 3.0\ nC$ is placed at $(1, 2, 3)m$ and a second charge $Q_2 = -4.0\ nC$ is placed at $(-1, 0, 4)m$. What is the electric field at an arbitrary point (x, y, z) ?

$$\vec{E}_1(x,y,z) = \frac{kQ_1}{\mathscr{r}_1^3}\vec{\mathscr{r}}_1 = \frac{27\langle x-1,\ y-2,\ z-3\rangle}{[(x-1)^2+(y-2)^2+(z-3)^2]^{3/2}}$$

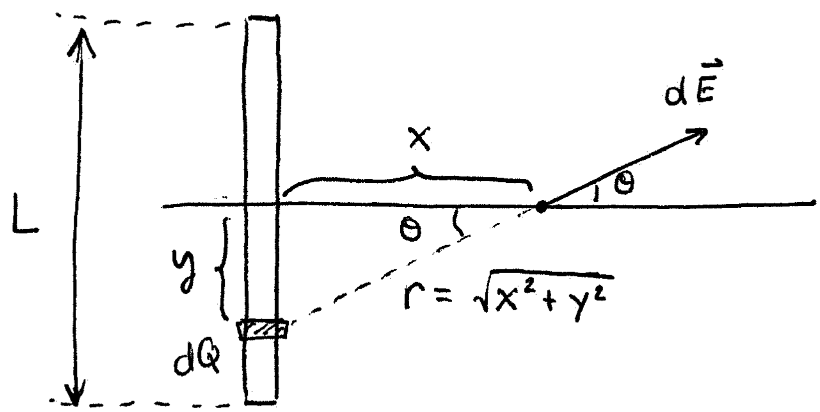
$$\vec{E}_2(x,y,z) = \frac{kQ_2}{\mathscr{r}_2^3}\vec{\mathscr{r}}_2 = \frac{-36\langle x+1,\ y,\ z-4\rangle}{[(x+1)^2+(y^2+(z-4)^2]^{3/2}}$$

(omitting units) (assuming x, y, z in meters)

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \left(\frac{27\langle x-1,\ y-2,\ z-3\rangle}{[(x-1)^2+(y-2)^2+(z-3)^2]^{3/2}} - \frac{36\langle x+1,\ y,\ z-4\rangle}{[(x+1)^2+y^2+(z-4)^2]^{3/2}}\right)\frac{N}{C}$$

Problem 4 A positive charge Q is evenly distributed from $(0, -L/2)$ to $(0, L/2)$. Find the electric field due to this charge distribution at $(x, 0)$ for $x > 0$.



$$\frac{dQ}{dy} = \frac{Q}{L}$$

for $-L/2 \le y \le L/2$

$$dE_x = \left(\frac{k\,dQ}{r^2}\right)\cos\theta = \frac{kQ\,dy}{L(x^2+y^2)} \cdot \frac{x}{\sqrt{x^2+y^2}}$$

By symmetry,

$$E_x = 2\int_0^{L/2} \frac{kQx}{L} \frac{dy}{(x^2+y^2)^{3/2}}$$

$$= \frac{2kQ}{L}\int_0^{L/2} \frac{x\,dy}{(x^2+y^2)^{3/2}}$$

$y = x\tan\theta$

$x^2+y^2 = x^2\sec^2\theta$

$dy = x\sec^2\theta\,d\theta$

$$= \frac{2kQ}{L}\int_{y=0}^{y=L/2} \frac{x^2\sec^2\theta\,d\theta}{(x^2\sec^2\theta)^{3/2}}$$

$$= \frac{2kQ}{Lx}\int_{y=0}^{y=L/2} \cos\theta\,d\theta$$

$$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$

$$= \frac{2kQ}{Lx} \left.\frac{y}{\sqrt{x^2+y^2}}\right|_{0=y}^{L/2=y}$$

$$= \frac{2kQ}{Lx} \frac{L/2}{\sqrt{x^2+L^2/4}}$$

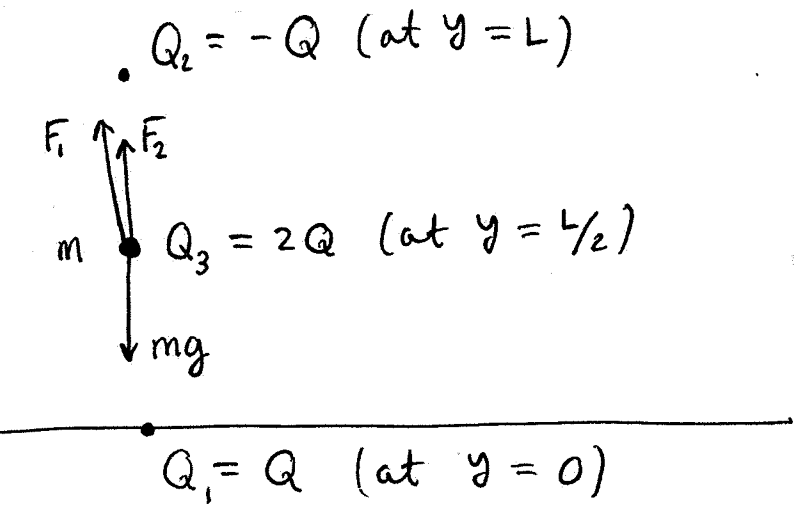
$$= \boxed{\frac{kQ}{x\sqrt{x^2+L^2/4}}}$$

directed right

$\vec{E} = \langle E_x, 0\rangle$ since $E_y = 0$ by symmetry.

Problem 5 Suppose $Q_1 = Q$ is stuck at $y = 0$ and $Q_2 = -Q$ is stuck at $y = L$. Assume these charges are near the surface of the Earth and $F_g = -mg\hat{y}$. If a charge $Q_3 = 2Q$ is in equilibrium at $y = L/2$ between Q_1 and Q_2 then what is the mass m of the middle charge ? Include a free body diagram for the middle charge including all the gravitational and electric forces which act on m. (all three charges are placed where $x = 0$)

$$\underbrace{F = \frac{kQ_1Q_2}{r_{12}^2}}_{\text{Coulomb Force}}$$

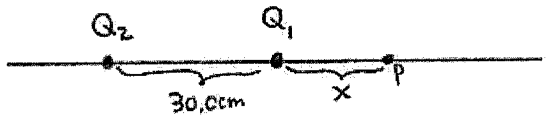


$$ma = 0 = \underbrace{\frac{kQ(2Q)}{(L/2)^2}}_{F_1} + \underbrace{\frac{k(Q)(2Q)}{(L/2)^2}}_{F_2} - mg$$

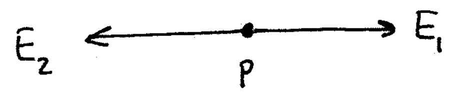
$$mg = \frac{4kQ^2}{L^2/4} = \frac{16kQ^2}{L^2} = \frac{16Q^2}{4\pi\epsilon_0 L^2}$$

$$\boxed{m = \frac{4Q^2}{\pi\epsilon_0 g L^2}} \quad \text{or} \quad \boxed{m = \frac{16kQ^2}{gL^2}}$$

Problem 6 Suppose a charge $Q_1 = 4.0\ nC$ has a charge $Q_2 = -60.0\ nC$ are placed distance $30.0\ cm$ apart. Find x such that the electric field at P is zero.



$$\ell = 30.0\ cm$$
$$Q = 4.0\ nC$$
$$Q_1 = Q, \quad Q_2 = -15Q$$



$$E_2 = \frac{kQ_2}{(x+\ell)^2} \quad \text{and} \quad E_1 = \frac{kQ_1}{x^2}$$

For $E = 0$ at P we need $E_1 = E_2$

$$\frac{15kQ}{(x+\ell)^2} = \frac{kQ}{x^2}$$
$$(x+\ell)^2 = 15x^2$$
$$x^2 + 2x\ell + \ell^2 = 15x^2$$
$$-14x^2 + 2x\ell + \ell^2 = 0$$

$$x = \frac{-2\ell \pm \sqrt{4\ell^2 - 4(-14)\ell^2}}{2(-14)}$$

$$x = \left(\frac{1 \pm \sqrt{15}}{14}\right)\ell$$

$$x \approx 0.3481\ell \quad \text{or} \quad \underbrace{-0.2052\ell}_{\text{unphysical}}$$

$$\boxed{x = 10.44\ cm}$$