Print Names in Team: ____

PHYSICS 232	MISSION 5: MAGNETOSTATICS

Please work each problem in the white space provided. Attach additional sheets if necessary. Print this one-sided and staple in the top left corner with a metal staple once complete. Each team turns in one document.

Problem 28 Find the electromagnetic force on a charge $Q = 3.0\ nC$ which moves at velocity $\vec{v} = \langle 10, 20, 4 \rangle\ m/s$ through a space with electric field $\vec{E} = \langle 2.0, -10, 0 \rangle\ N/C$ and magnetic field $\vec{B} = \langle 1, 2, 3 \rangle\ T$.

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$= (3.0nC)\left[\langle 2.0, -10.0, 0 \rangle \frac{N}{C} + \langle 10, 20, 4 \rangle \times \langle 1, 2, 3 \rangle \frac{mT}{s} \right]$$

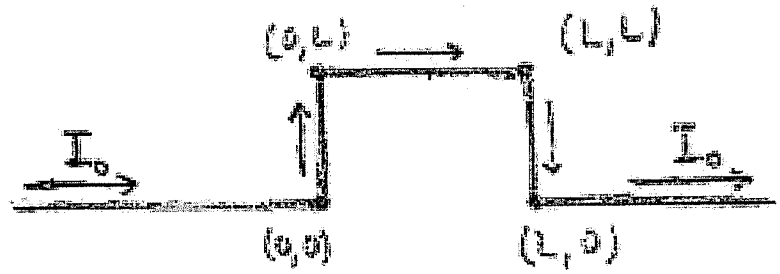
$$= (3.0nC)\left[\langle 2.0, -10.0, 0 \rangle \frac{N}{C} + \langle 60-8, 4-30, 20-20 \rangle \frac{N}{C} \right]$$

$$= (3.0nC)\left[\langle 2, -10, 0 \rangle + \langle 52, -26, 0 \rangle \right] \frac{N}{C}$$

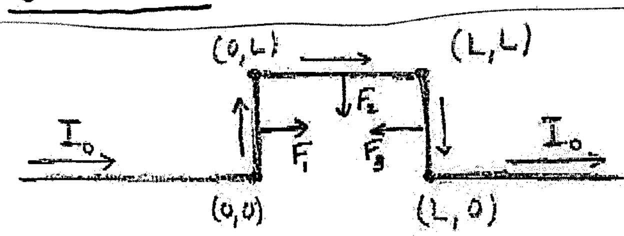
$$= (3.0n)\left[\langle 54, -36, 0 \rangle \right] N$$

$$= \langle 1.62, -1.08, 0 \rangle 10^{-7} N = \boxed{\langle 162nN, -108nN, 0 \rangle}$$

Problem 29 Suppose a rectangular half-loop is formed by a long-wire which has a current flowing from $(-\infty, 0)$ to $(0,0)$ to $(0,L)$ to (L,L) and back to $(L,0)$ and on to $(\infty, 0)$. If a magnetic field is applied such that it comes out of the page on which the loop is drawn then what is the magnetic force placed on the loop?



SOLUTION:



$\vec{F}_1 + \vec{F}_3 = 0$ by symmetry.

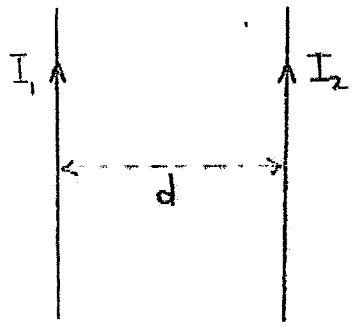
we need only calculate $\vec{F}_2$.

$$\vec{F}_2 = I_0 \vec{l} \times \vec{B}$$

$$= (I_0 L B_0)(-\hat{j}) \text{ since } \vec{l} \perp \vec{B} \quad \left(\vec{l} = L\hat{i}, \ \vec{B} = B_0\hat{k} \ / \ \vec{l} \cdot \vec{B} = 0 \right)$$

$$\Rightarrow \boxed{\vec{F}_2 = -I_0 L B_0 \hat{j}}$$

Problem 30 Two long wires carrying currents $I_1 = 2.5\ mA$ and $I_2 = 4.0\ mA$ are placed parallel to one another a distance $d = 0.25\ m$ apart. If both currents flow in the same direction then what is the force per unit length on the currents ? Does this force push the wires apart or does it pull them together ?



Consider a particular line-segment,

$$\|\vec{F}_2\| = \|l\vec{I}_2 \times \vec{B}_1\| = l\left(\frac{\mu_0 I_2 I_1}{2\pi d}\right)$$

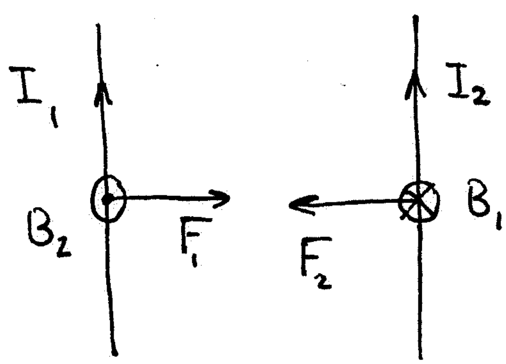
$$\|\vec{B}_1\| = \frac{\mu_0 I_1}{2\pi d}$$

$$\Rightarrow \quad \frac{F_2}{l} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{(4\pi\times10^{-7})(2.5m)(4.0m)}{2\pi\,(0.25)}\ \frac{N}{m}$$

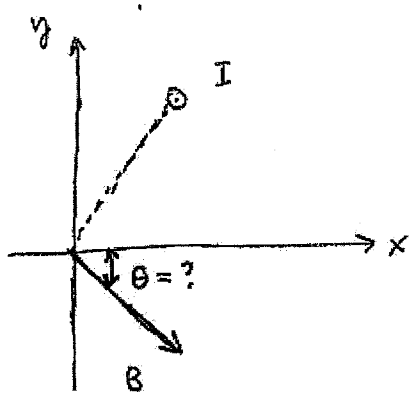
By $1 \leftrightarrow 2$ interchange symmetry,

$$\Rightarrow \quad \boxed{\frac{F}{l} = 8\times10^{-12}\ \frac{N}{m}}$$

It pulls the currents together



Problem 31 A current of 2.5 A flows directly out of the page at $(1,2)m$. Find the magnitude and direction of the magnetic field $\vec{B}$ at the origin $(0,0)$ due to the given current.

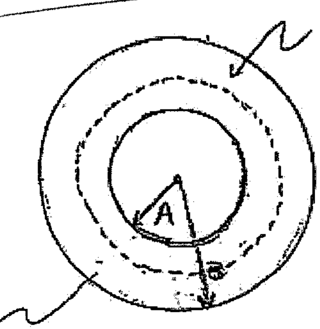


$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\frac{Tm}{A}\right)(2.5A)}{2\pi\sqrt{1+4}\ m} = 2.24\times10^{-7}\ T$$

$$\theta = \tan^{-1}\left(\frac{2}{1}\right) - 90° = -26.5° \text{ (see picture)}$$

$\therefore$ $B = 2.24\times10^{-7}\ T$ at $\theta = 333.5°$

Problem 32 Consider a uniform current I_o which flows parallel to the z-axis from inner radius A to an outer radius B from the z-axis. Find the magnitude of the magnetic field a function of the distance r to the z-axis.



Amperian Loop at $A \le r \le B$ encloses $\left(\dfrac{\pi r^2 - \pi A^2}{\pi B^2 - \pi A^2}\right) I_0$ current.

(I_0 flows throughout this $A \le r \le B$ region)

By cylindrical symmetry,

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 I_0 \left(\frac{r^2 - A^2}{B^2 - A^2}\right)$$

$$\boxed{B = \frac{\mu_0 I_0 (r^2 - A^2)}{2\pi r (B^2 - A^2)} \quad \text{for } A \le r \le B}$$

Then $\boxed{B = 0 \quad \text{for } 0 \le r \le A}$ and of course

$\boxed{B = \dfrac{\mu_0 I_0}{2\pi r} \quad \text{for } r \ge B}$ since any Amperian loop with $r \ge B$ encloses the whole current I_0.