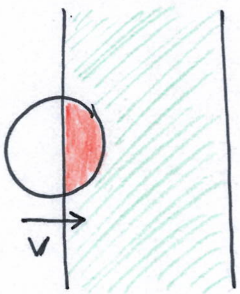
Print Names in Team: ______________________________________.

PHYSICS 232 — MISSION 6: FARADAY'S LAW & APPLICATIONS

Please work each problem in the white space provided. Attach additional sheets if necessary. Print this one-sided and staple in the top left corner with a metal staple once complete. Each team turns in one document.

Problem 33 Suppose a magnetic field is zero everywhere except for a region $0 \leq x \leq L$ where a magnetic field of $B = 2.0\ T$ directed in the positive z-direction (out of the page). In other words, $\vec{B} = (2.0T)\hat{z}$ for $0 \leq x \leq L$ and $\vec{B} = 0$ elsewhere. Suppose a loop of wire travels in the positive x-direction in the xy-plane. Find the following: (assume the loop is a circle of radius less than L)

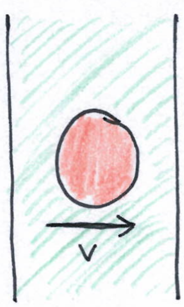
(a.) Find the direction of the induced current in the loop as the loop enters the region $0 \leq x \leq L$



flux upward ($\hat{z}$) increasing

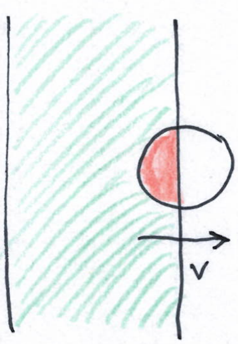
$\Rightarrow$ CW induced current

(b.) Find the magnitude of the induced current in the loop as the loop is inside the region $0 \leq x \leq L$



$$\mathcal{E} = -\frac{d\Phi_B}{dt} = 0 \Rightarrow \boxed{I = 0}$$

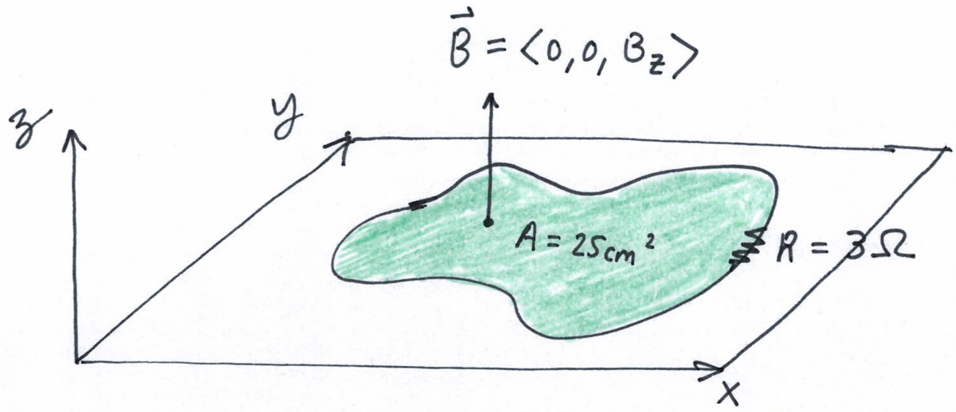
(c.) Find the direction of the induced current in the loop as the loop leaves the region $0 \leq x \leq L$



flux upward ($\hat{z}$) decreasing

$\Rightarrow$ CCW induced current.

Problem 34 Suppose $B_z(t) = \alpha \sin(kt)$ is the z-component of the magnetic field in the xy-plane where $\alpha = 2.0\ T$ and $k = 10\ Hz$. This means the magnitude changes the same way at all points in the plane. Suppose a $25\ cm^2$ loop with resistance $3.0\ \Omega$ is placed in the xy-plane. What current is induced in the loop at time t ?



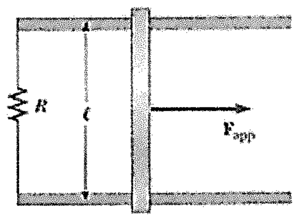
$$\Phi_B = \iint \vec{B} \cdot d\vec{S} = B_z A = A\alpha \sin(kt)$$

$$\mathcal{E} = \frac{-d\Phi_B}{dt} = -A\alpha k \cos(kt) = IR$$

$$\therefore \quad \boxed{I(t) = \frac{-A\alpha k \cos(kt)}{R}}$$

$$\left(I(t) = \frac{-(2.0T)(25cm^2)(10\ \frac{1}{s}) \cos(10t/s)}{3\Omega} \right)$$

Problem 35 The figure below shows a top view of a bar that can slide without friction. The resistor is 6.30 Ω and a 2.50 T magnetic field is directed perpendicularly downward, into the paper. Let $\ell = 1.20\ m$.



$$A = x\ell$$

$$\Phi_B = BA = Bx\ell$$

$$\frac{d\Phi_B}{dt} = B\ell\frac{dx}{dt} = B\ell v = |\mathcal{E}| \Rightarrow \underline{I = \frac{B\ell v}{R}}$$

(a.) Calculate the applied force required to move the bar to the right at a constant speed of 1.90 m/s.

$$\|\vec{F}_{wire}\| = \|\underbrace{I\vec{\ell}\times\vec{B}}_{\text{LEFTWARD}}\| = \left(\frac{B\ell v}{R}\right)\ell B \quad \text{since } \vec{\ell} \perp \vec{B}$$

$$ma = 0 = F_{pushing} + F_{wire} \Rightarrow F_{pushing} = \frac{B^2\ell^2 v}{R}$$

$$\therefore F_{applied} = \frac{(2.50)^2(1.20)^2(1.90)}{6.30}\ N = \boxed{2.714\ N}$$

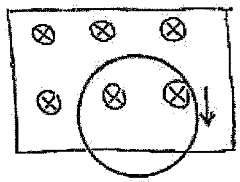
(b.) At what rate energy delivered in the resistor ?

$$P = I^2R = \left(\frac{B\ell v}{R}\right)^2 R = \frac{B^2\ell^2v^2}{R} = \left(\frac{B^2\ell^2 v}{R}\right)v$$

$$\boxed{P = 5.157\ W}$$

Problem 36 Given the diagrams below, indicate the direction of the induced current in each case:

(a.) .



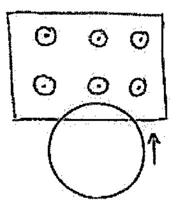
flux into page <u>decreasing</u>
as the loop moves down

⇒ induced current should <u>increase</u> downward flux

⇒ $\boxed{\text{CW current induced}}$

i_{induced}

(b.) .

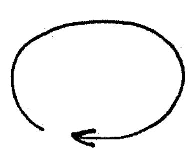


flux out of page <u>increasing</u>
as the loop moves up

⇒ induced current should <u>decrease</u> upward flux

⇒ induced current should increase
downward flux

⇒ $\boxed{\text{CW current induced}}$



i_{induced}

Problem 37 A solenoid of length 20.0 cm with N-turns has a current $I = 6.00\ A$ flowing. If the magnetic field strength near the center of the solenoid is measured to have a magnitude 0.0188 T then what is N ? Assume the edge-effects are neglible.

$$B = \mu_0 n I$$ (by Ampere's Law as we saw in lecture)

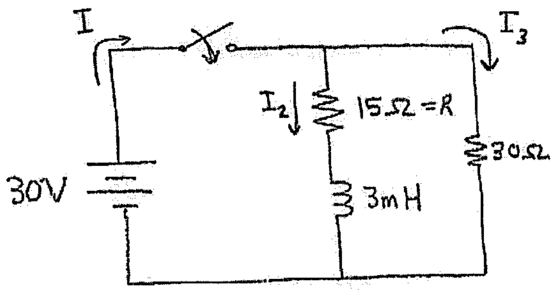
$$n = \frac{N}{L}$$

We wish to find N, we're given $L = 20\,cm = 0.2\,m$

$$B = \frac{\mu_0 N I}{L}$$

$$\Rightarrow N = \frac{BL}{\mu_0 I} = \frac{(0.0188T)(0.2m)}{(4\pi \times 10^{-7}\frac{Tm}{A})(6.00A)} = \boxed{498.7}$$

Problem 38 Find the current as a function of time for the RL-circuit pictured below for $t > 0$. Assume the pictured switch is closed at time $t = 0$.



$$I = I_2 + I_3 \quad , \quad I_3 = \frac{30\,V}{30\,\Omega} = 1.0\,A$$

$$\mathcal{E}' - I_2 R - L\frac{dI_2}{dt} \quad , \quad I_2(0) = 0$$: current zero before the switch closed.

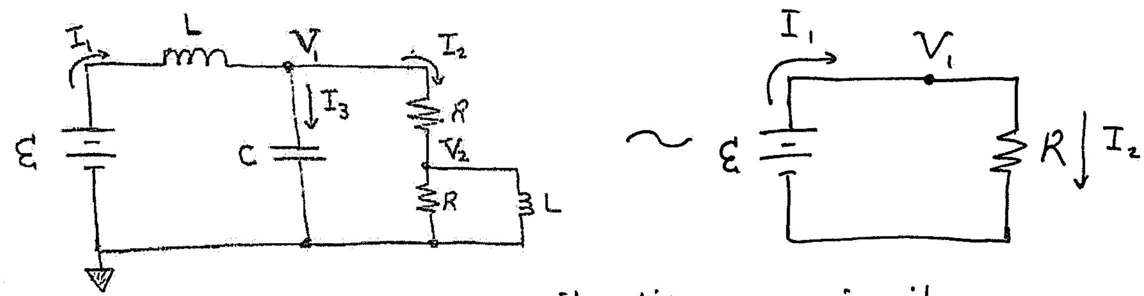
$$\frac{dI_2}{dt} + \frac{R}{L}I_2 = \frac{\mathcal{E}'}{L} \Rightarrow I_2(t) = c_1 e^{-t/\tau} + \frac{\mathcal{E}'}{R} \quad \text{where } \tau = \frac{L}{R}$$

Note $I_2(0) = c_1 + \frac{\mathcal{E}'}{R} = 0$

Thus $I_2(t) = \frac{\mathcal{E}'}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

$$\therefore \boxed{I = \left[1 + 2\left(1 - e^{-5000t/s}\right)\right]A}$$

Problem 39 Find the currents and voltages indicated below (assume the circuit has been connected a long time)



Capacitor like open circuit
inductor like short circuit

$I_1 = \mathcal{E}/R$

$I_2 = \mathcal{E}/R$

$I_3 = 0$ (capacitor like open circuit)

$V_1 = \mathcal{E}.$

$V_2 = 0.$

Problem 40 Write Maxwell's Equations in both integral and differential form (name each one).

$$\int_S \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \longleftrightarrow \nabla \cdot \vec{E} = \rho/\epsilon_0 \quad \text{(Gauss' Law)}$$

$$\int_S \vec{B} \cdot d\vec{A} = 0 \longleftrightarrow \nabla \cdot \vec{B} = 0 \quad \text{(no magnetic monopoles)}$$

$$\int_{\partial S} \vec{B} \cdot d\vec{l} = \mu_0 \int_S \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \cdot d\vec{A} \longleftrightarrow \nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \quad \text{(Ampere's Law with Maxwell's Correction)}$$

$$\int_{\partial S} \vec{E} \cdot d\vec{l} = \frac{d}{dt} \int_S \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt} \longleftrightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday's Law)}$$

Problem 41 Show that Maxwell's Equations imply the local conservation of charge.

Consider a volume V,

$$Q_V = \iiint_V \rho \, dV$$

$$\frac{dQ_V}{dt} = \frac{d}{dt} \iiint_V \rho \, dV = \frac{d}{dt} \iiint_V \epsilon_0 (\nabla \cdot \vec{E}) \, dV = \epsilon_0 \frac{d}{dt} \iint_{\partial V} \vec{E} \cdot d\vec{A} = \epsilon_0 \iint_{\partial V} \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

rate of charge flowing out of V

$$= \iint_{\partial V} \vec{J} \cdot d\vec{A} = \iint_{\partial V} \left(\frac{\nabla \times \vec{B}}{\mu_0} - \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{A}$$

$$= \iiint_V \frac{1}{\mu_0} \underbrace{\nabla \cdot (\nabla \times \vec{B})}_{0} - \epsilon_0 \iint_{\partial V} \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

$$= -\epsilon_0 \iint \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A} = -\frac{dQ_V}{dt}$$

(charge leaving the volume is same as reduction in net-charge in the volume)

That is, $\frac{d}{dt} \iiint_V \rho \, dV = -\iint_{\partial V} \vec{J} \cdot d\vec{A} = -\iiint_V (\nabla \cdot \vec{J}) \, dV \Rightarrow \boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$